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XXXIII. *Flexure of long Pillars under their own Weight.* *By* MAURICE F. FITZGERALD*.

THE origin is taken at the upper end of the neutral axis, abscissæ being reckoned positive vertically downwards, and ordinates horizontal. The flexure is supposed small, and assumed to lie in a vertical plane. The symbols employed are as follows:—

H = total height of pillar;

h = height below top of any point in it;

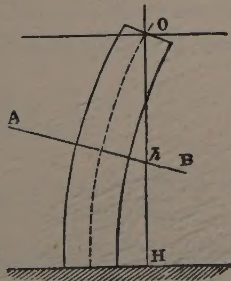
S = total shear on a section normal to neutral axis;

M = bending-moment;

w = weight of pillar per unit of length;

E and I, as usual, stand for the coefficient of flexural elasticity, and moment of inertia of cross section, respectively.

Fig. 1.



Taking a plane, A B (fig. 1), normal to the neutral axis, the shear on this plane is the component along it of the weight of the upper part of the pillar (whose top is supposed free); for small bending we have therefore

$$S = wh\frac{dy}{dh} \text{ nearly.}$$

* Read February 26, 1892.

By well-known theorems, $\frac{dM}{dh} = S$ and $M = -EI\frac{d^2y}{dh^2}$, which give by substitution,

$$EI\frac{d^3y}{dh^3} = -wh\frac{dy}{dh}.$$

By writing $\frac{h}{H} = x$ and $m = \frac{wH^3}{EI}$, this takes the form

$$\frac{d^3y}{dx^3} = -mx\frac{dy}{dx},$$

in which, putting $\frac{dy}{dx} = u$, we get

$$\frac{d^2u}{dx^2} = -m\,xu,$$

a differential equation which enters into other questions.

The value of $x\left(=\frac{h}{H}\right)$ runs from 0 at top to 1 at foot of pillar; m has, except for pieces of fine wire a few feet in length, or for very unusually tall and large columns, only a small fractional value in practice.

Integrating the equation $\frac{d^3y}{dx^3} = -mx\frac{dy}{dx}$ in series, we get

$$y = AU + BV,$$

where

$$U = x\left\{1 - \frac{mx^3}{2.3.4} + \frac{m^2x^6}{2.3.5.6.7} - \frac{m^3x^9}{2.3.5.6.8.9.10} + \ldots\right.$$

V is another series, having x^2 as a factor, and A and B are arbitrary constants.

Calling the first derived function, with respect to x, of U, U′ and so on, the condition of a pillar free at top, and fixed initially vertically to a rigid base is expressed by

$$\frac{dy}{dx} = AU' + BV' = 0 \text{ when } x = 1, \textit{ i. e.} \text{ at foot,}$$

and

$$\frac{d^2y}{dx^2} = AU'' + BV'' = 0 \text{ when } x = 0, \textit{ i. e.} \text{ at top,}$$

since there is no bending-moment at top.

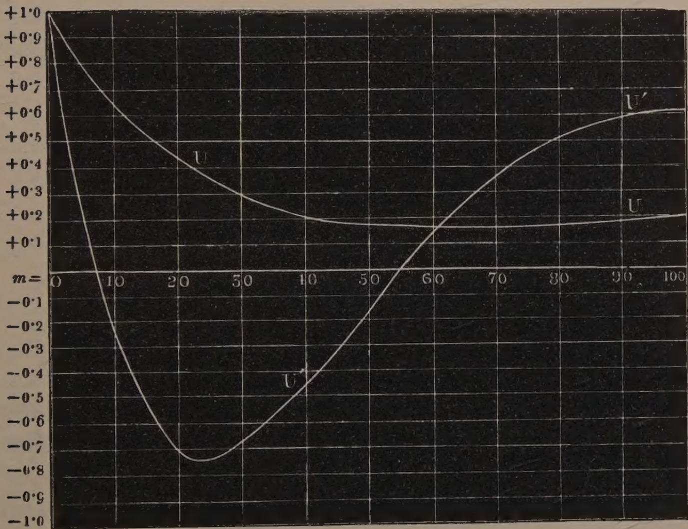
As V contains x^2 as a factor, the second of these gives $B = 0$,

and the first then requires $U'=0$ when $x=1$. It will be found, on inspecting the curves plotted in fig. 2, that a value

Fig. 2.

$$y=U_{x=1} \quad \frac{d^3y}{dx^3}=-mx\frac{dy}{dx},$$

$$u=U'_{x=1} \quad \frac{d^2u}{dx^2}=-mxu.$$



of $m=7{\cdot}85$ nearly is that required. For dimensions in feet, and for steel in which E = about 12,000 tons per square inch, this gives, on putting in the numerical values, and putting L = ratio of length to diameter,

$$H \text{ (in feet)} = \frac{8{\cdot}1 \times 10^6}{L^2} \text{ for steel tubes,}$$

H being here independent of the thickness, supposed small; and

$$H = \frac{4 \times 10^6}{L^2} \text{ for round steel rods,}$$

as the limiting height of pillar which can stand without bending under its own weight. Thus for $L=100$, the maximum height is about 800 feet, giving a tube 8 feet diameter. For wires, L may be much greater; for instance, the limit at which bending due to its own weight, of wire originally

straight and vertical, size No. 28 B.W.G., must occur is about 1·8 feet.

All columns, in practice, naturally fall far within the limits here given. In connexion, however, with the inherent flexibility of very large masses under their own weight, even when direct crushing is prevented (say by external fluid pressure), it may be remarked that for $L = 4$, $H = 47$ miles, approximately; so that a solid steel column 12 miles diameter would bend, even if prevented from bulging, if it were 50 miles high.

The only case of interest, besides that of a column fixed at its base and free at the top, above treated, seems to be that of a heavy upright column, held at top and bottom by external bending-moments so that the neutral axis is vertical at both ends, but otherwise free.

In this case, denoting by suffixes the values at each end, we have

$$AU_0' + BV_0' = 0, \qquad AU_1' + BV_1' = 0,$$

$$AU_0'' + BV_0'' = M_1H^2, \quad AU_1'' + BV_1'' = M_2H^2.$$

V_0' and U_0'' are both zero identically; $U_0' = 1$, and $V_0'' = 2$, which give $A = 0$; $2B = M_1H^2$; and, on substitution in the second and last of the above equations, we get

$$BV_1' = 0 \text{ and } M_1V_1'' = 2M_2,$$

where, in V_1'', the value of m which makes $V_1' = 0$ is to be inserted. The result shows that there is, in this case, a definite ratio between the external bending-moments.

Precisely similar results, as to producing bending, would take place in a bar accelerated by a force applied at its back end, neglecting longitudinal sound-waves; as also to a liquid filament retarded, if it possessed uniform stiffness in virtue of any internal motion.

Belfast, February 1, 1892.

Since the above was in print Prof. A. G. Greenhill, F.R.S., has sent to the writer a paper, published in the Proc. Camb. Phil. Soc. vol. iv. 1881, written by him for Prof. Asa Gray, on the greatest height of poles, masts, and trees, consistent with stability. The differential equation involved is, in Prof.

Greenhill's paper, solved by the aid of Bessel's functions, and the investigation is extended to the cases of a solid cone and a paraboloid of revolution, the general form of the solution for certain other solids of revolution being given. The results for a wire (allowing for a slight difference in the value assumed for E) given by Prof. Greenhill are the same as those above. The function U′ tabulated in the curve fig. 2 appears, from Prof. Greenhill's paper, to be connected with $J_n(\kappa x^m)$ by the relation

$$U' = x^{\frac{1}{2}} J_{-\frac{1}{3}}(\kappa x^{\frac{3}{2}}).$$

Belfast, March 16, 1892.

XXXIV. *Choking Coils.*

By Professor John Perry, *D.Sc.*, *F.R.S.**

There is eddy-current loss of power in all the conducting masses of a choking coil. Hence a choking coil is really a transformer with one primary coil and many secondaries, and much magnetic leakage. In a transformer with many coils, whether or not they have magnetic leakage, it may be shown that any given group of secondaries of given numbers of turns and resistances may be replaced by one secondary without affecting the currents in the other coils; and we may take a choking coil to be a transformer with a primary coil of N turns and resistance R ohms, with C amperes flowing at any instant, the potential difference at its terminals being V, and a secondary coil closed on itself of n turns, resistance r ohms, and current c amperes.

If we assume that the induction per square centimetre β is the same everywhere, and if it follows the law

$$\beta \text{ in C.G.S. units} = \Sigma a_i \sin(ikt + e_i),$$

the average power in watts, wasted in eddy currents in the iron per cubic centimetre is

$$6{\cdot}25 \times 10^{-13} r^2 k^2 \Sigma\, i^2 a_i^2,$$

if the specific resistance of the iron is taken to be 10^4 C.G.S. units. It is less at higher temperatures, being inversely

* Read March 11, 1892.

proportional to the specific electric resistance of the iron. The iron is supposed to be of wire of radius r centimetres. Even when we leave the eddy-current loss in the copper out of account, it is to be remembered that the induction is not uniform in the section of a wire, nor is the average induction in each wire the same for all the wires, and therefore the real loss of power in the iron by eddy currents is always greater than the result of applying this formula.

I am going to assume that one secondary coil with no magnetic leakage may be substituted for all the eddy-current circuits, and this is the same as assuming the truth of the above rule. I ignore magnetic leakage because this is only a preliminary note, and such experiments as have hitherto been made do not enable me to take account of it, for there are no experimental measurements as yet of the want of uniformity of the induction.

The equations of the two circuits are

$$\mathrm{V}=\mathrm{RC}+\mathrm{N}\theta\mathrm{I} \quad \text{and} \quad 0=rc+n\theta\mathrm{I},$$

if I is the total induction (10^8 C.G.S. units being taken as the unit of induction). If

$$\mathrm{A}=\mathrm{NC}+nc \quad \text{and} \quad q=\mathrm{N}^2/\mathrm{R}^2+n^2/r^2$$

(the term n^2/r^2 being really negligible), the fundamental equation for calculating I is

$$\mathrm{A}+q\theta\mathrm{I}=\mathrm{NV/R}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Given the law connecting A and I and the resistances and V, I may be calculated, and consequently C and c. Now in ordinary practical transformer calculations A may be neglected in the equation, even with the most complex law of magnetization; and it is this that causes calculations of the induction and secondary currents and voltages in the most complex cases, and even the primary current, unless when there is a small load on the transformer, to be exceedingly easy even when the coils are curiously connected with condensers and choking coils, and when there is much magnetic leakage. But in our present case, it is A itself which is wanted, and another method of working must be adopted. In fact, the value of C does depend very much upon the law of magnetization.

However complicated the magnetic law may be, it can be expressed in the shape :—

If $$I=\Sigma A_i\sigma_i \sin ix,$$

then

$$A=\Sigma A_i\{\sin(ix+f_i)-b_i\sin 3ix+m_i\sin 5ix-\&c.\},$$

x being any quantity which increases continually. To be strictly accurate, even as well as odd harmonics of ix exist in A, and one of my students, Mr. Fowler, has worked them out for some of Prof. Ewing's curves ; but the above formula has been found by Mr. Field to be sufficiently accurate. Of course, instead of ix we may have $(ix+e_i)$ in the above general expression.

When there is no hysteresis, $f_i=0$. When there is constant permeability (no hysteresis and no saturation), not only is $f_i=0$, but $b_i=0$, $m_i=0$ &c., and $\sigma_i=\sigma_1$.

If μ the magnetic permeability of the iron is constant and the magnetic circuit is altogether of iron, as it always ought to be both in transformers and in choking coils, σ_1 stands for $4\pi a\mu 10^{-9}/\lambda$, where a is the area of cross section of the iron in square centimetres, and λ is the average length of the induction solenoids in centimetres,

Equation (1) becomes in the most general case

$$NV/R=\Sigma A_i\{\cos f_i \sin(ikt+e_i)+(\sin f_i+qik\sigma_i)\cos(ikt+e_i)$$
$$-b_i\sin 3(ikt+e_i)+m_i\sin 5(ikt+e_i)\};$$

and hence, if V or I is given as a periodic function of the time, the other can be found and A and therefore c or C.

If V is a simple sine function of the time, I is so also, with very great, but not perfect accuracy. Assuming that I is a simple sine function, the neglected terms in V can now be calculated. The only problem, however, of importance is the calculation of C assuming that V follows the law $V=V_0\sin kt$.

We may take $q=N^2/R$. Hence

$$-I=(V_0/Nk)\cos kt$$

very nearly, and if $e=n^2\sigma k/r$, being called the eddy-current effect, f being the hysteresis term,

$$C=V_0\Big[(1+2e\sin f+e^2)^{\frac{1}{2}}\sin\Big\{kt-90^\circ+\tan^{-1}\Big(\tan f+\frac{e}{\cos f}\Big)\Big\}-$$
$$b\cos 3kt-m\cos 5kt\Big]\div N^2\sigma k\,. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

We see that the effect of eddy currents without hysteresis is to increase the amplitude of the important term in C, and to produce a lead of $90^\circ-\cot^{-1}e$, whereas the effect of hysteresis without eddy currents is to keep the amplitude unaltered and to produce a lead f. If f is put equal to 0, that is if we assume no hysteresis, we obtain results which seem to be in accordance with such experimental observations as have yet been made.

The effective current $\overline{C}$ (if $\overline{V}$ is the effective voltage), with constant permeability, is $\overline{C}=\overline{V}/N^2\sigma k$. With hysteresis (or with no hysteresis but some saturation of the iron), but no eddy currents, $\overline{C}=1{\cdot}02\,\overline{V}/N^2\sigma k$, taking b as $\cdot 2$.

With eddy currents and hysteresis,

$$\overline{C}=\overline{V}\sqrt{1{\cdot}04+2e\sin f+e^2}/N^2\sigma k.$$

The average power given to the choking coil or average value of V C is

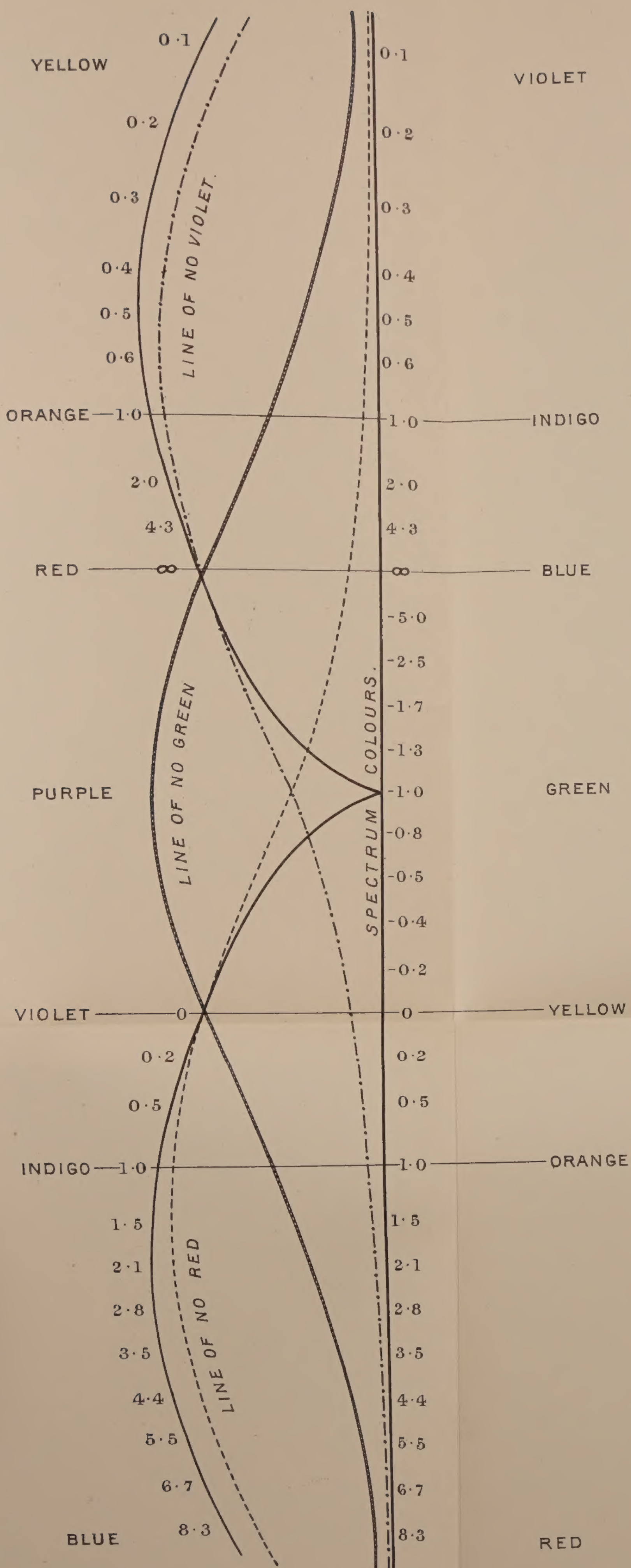
$$\overline{V}\,\overline{C}\,(e+\sin f)/(1+e^2+2e\sin f),$$

neglecting the small terms due to b and m, and this may be done in all cases where there is not much saturation.

Probably there are always traces of the terms in $3kt$ and the higher harmonics in both V and I, but they must certainly exist in either V or I even when there is not much saturation.

It almost seems that in a choking coil we have found what has long been looked for, a method of increasing frequency by mere magnetic means. A condenser shunting a non-inductive part of the circuit would receive currents in which the higher harmonics would be greatly magnified in importance.

To show the magnitude of the terms in (2) I will take a well-known 1500-watt transformer, unloaded, as a choking coil. Here $q=7837$. The total average power wasted in heating the iron being 40 watts, I assume that this is altogether due to eddy currents. Power wasted in eddy currents being $n^2V_0^2/2rN^2$, we have $n^2/r=2{\cdot}117$, when $V_0=2828$. An eddy-


YELLOW
VIOLET
ORANGE
INDIGO
RED
BLUE
PURPLE
GREEN
VIOLET
YELLOW
INDIGO
ORANGE
BLUE
RED
LINE OF NO VIOLET.
LINE OF NO GREEN
LINE OF NO RED
SPECTRUM COLOURS.
0·1
0·2
0·3
0·4
0·5
0·6
1·0
2·0
4·3
∞
-5·0
-2·5
-1·7
-1·3
-1·0
-0·8
-0·5
-0·4
-0·2
0
0·2
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2·1
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3·5
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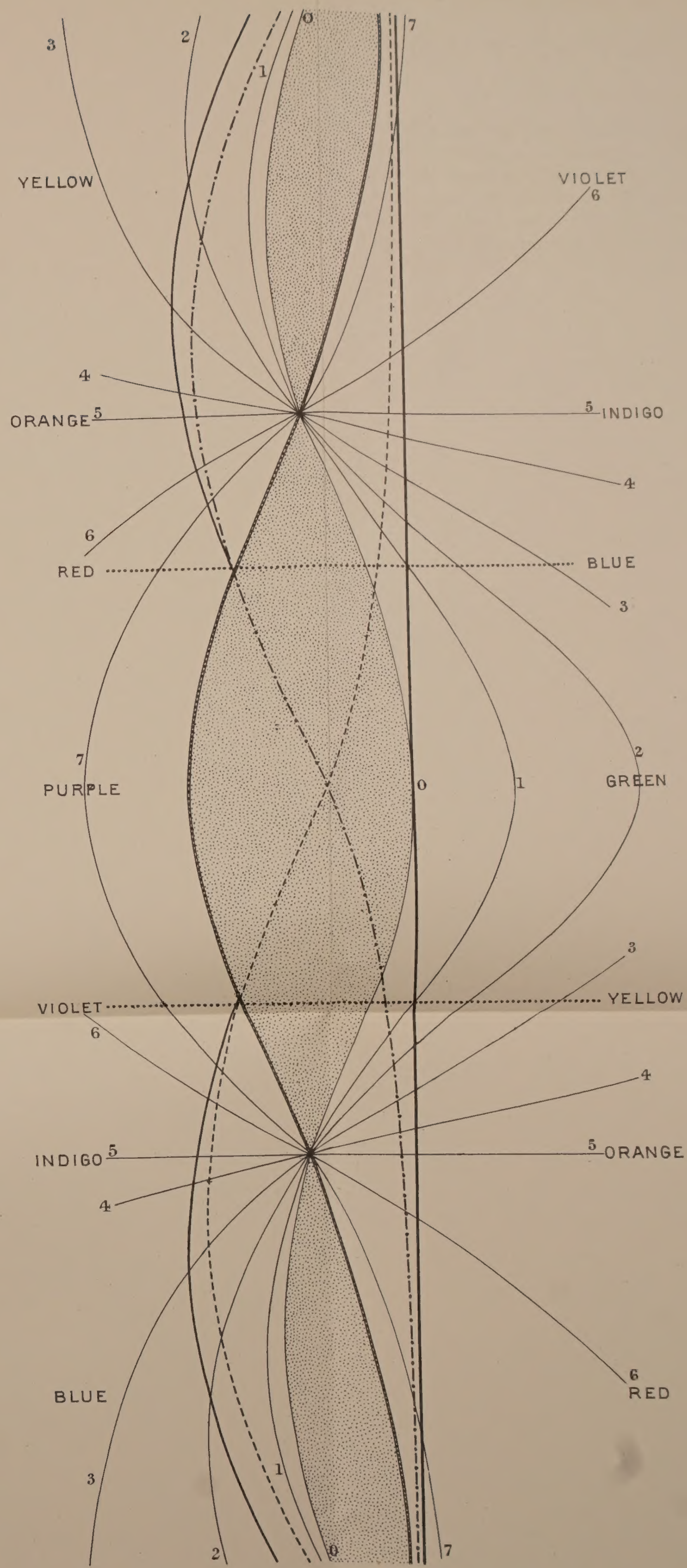
6·7
8·3

Fig. 4.


YELLOW
VIOLET
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VIOLET
YELLOW
INDIGO
ORANGE
BLUE
RED

Fig. 5.

current coil which would replace all the eddy-current circuits is a coil of 2 turns whose resistance is about 1·9 ohms, short-circuited on itself.

$$e=0·38, \text{ if } k=600.$$

It is obvious that e is proportional to k and to the square of the radius of the iron wire.

Assuming constant permeability and no eddy currents,

$$C=·074 \sin (kt-90°).$$

With some saturation but no hysteresis,

$$C=·079 \sin (kt-69°·2)-·0148 \cos 3kt-·0037 \cos 5kt,$$

if $b=0·2$, $m=·05$.

These values of b and m are usually employed by me for such magnetizations as are common in transformers. When I assume the existence of hysteresis, I take f about 20 degrees.

XXXV. *On the Construction of a Colour Map.* *By* Walter Baily, *M.A.**

[Plate XI.]

By the term Colour Map I mean a diagram each point of which defines by its position some particular colour. Such a colour map was designed by Clerk Maxwell in the form of a triangle, the angles of which were occupied by certain colours, and all other colours were treated as mixtures of these three primary colours, the composition of the mixture for the colour which occupied any particular point in the triangle being indicated by the length of the perpendiculars from that point on the sides of the triangle.

Now trilinear coordinates, although they afford very elegant methods for the solution of certain problems, are by no means so generally useful or so intelligible as the ordinary rectangular coordinates; and the fact that every colour can be defined by means of a spectrum colour and white light suggested to me the construction of a colour map with rectangular coordinates, in which measurement in one direction should indicate the wave-length of the spectrum colour employed,

* Read April 8, 1892.

and measurement at right angles to it should indicate the quantity of white light employed in defining the colour.

Let us take a vertical line to represent the spectrum, the lower end giving the red of the spectrum and the upper the violet. The spectrum is supposed to be formed so that equal differences of length measured along the spectrum represent equal differences in the wave-length; and when the quantity of colour at any point of the spectrum is mentioned, it is intended that a definite small part of the spectrum about that part is to be taken. Now all colours, except the purples, can be formed by adding white light to a spectrum colour. Let the amount of white light required be indicated by a line measured horizontally to *the right* from the proper point in the spectrum. Then the given colour is indicated by the point at the extremity of that line. Again, every colour except the greens has the following property: viz. that if it is added in the proper quantity to some spectrum colour, white is produced. Let the quantity of white produced be indicated by a line drawn from the proper point horizontally to the left. The point at the extremity of this line indicates the given colour. In this way a map is obtained in which every colour has its appropriate position. The greens occur only on the right hand, and the purples only on the left hand, but all other colours, as they can be indicated in both ways, occur on both sides of the spectrum line.

In using the term quantity of white light, I mean that a beam of white light is to be obtained in some definite manner from a definite source of light which forms the spectrum, and that the map is to show how much of this beam is used. Captain Abney finds that the positive pole of the electric arc is a source of light of constant quality, and uses it in his measurements; and he indicates the quantity of white light used by the ratio between its luminosity and that of the spectrum colour. It is a more complicated matter to express such a ratio than to express the amount of white light only, and I failed to work into a map Captain Abney's method of defining the quantity of white light.

The principle on which this map is founded will come out more clearly by the consideration of fig. 1, which may be considered as a sort of colour staff, to borrow a term from music.

The three horizontal lines represent the three colour sensations—Red, Green, and Violet, with such luminosity that the mixture represented by equal lengths of the three lines represents white light. Thus the vertical lines A, A′, wherever they may be placed, will include between them white light, which will be the more intense the farther they are apart. Any colour whatever may be represented by taking the line A as a base and measuring off the quantities of the sensations to points, R, G, and V. The distances included between A′

Fig. 1.

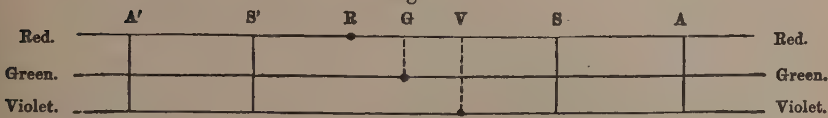


Fig. 2.

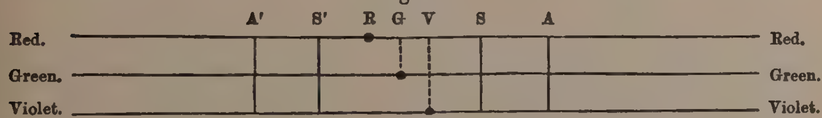
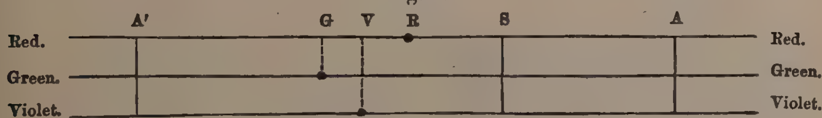


Fig. 3.



and R, G, V give every complementary colour to that represented by R, G, V, and A. The whole of the colours of this system are related together only by the position with respect to one another of R, G, V—that is, only by the differences R G and G V. But if we express the same colours at (say) half the luminosity, we must reduce all these distances to one half, as in fig. 2, and so with any other proportion. It is then not the differences R G and G V, but the ratio of these differences which is constant for all this system of colours. Hence, to determine to what system a colour belongs of which we know r, g, v, the quantities of red, green, and violet sensations respectively, we have only to obtain $\frac{r-g}{g-v}$.

In fig. 4, Pl. XI., the vertical line called "spectrum colours" is that along which the spectrum is thrown; and the lines

called "line of no Red," "line of no Green," and "line of no Violet" are lines to which distances are to be measured horizontally from any point to show the quantity of red, green, and violet sensations in the colour represented at that point. When these distances are measured from points on the spectrum line, they give the amount of such sensation for the corresponding spectrum colour. The curves which I have used are not intended to represent the true form of such curves, as it is sufficient for explaining the principles of the map that they should be curves having a maximum and shading off on each side. The numbers marked along the "spectrum line" give the value of the fraction $(r-g)/(g-v)$ at each point; and it will be seen that the value is large at the red end of the spectrum, probably beginning with infinity, and diminishes to zero, where the red and green are equal. It then changes sign and remains negative until g and v become equal, when the fraction becomes infinite and again changes sign. For the remainder of the spectrum the fraction continues positive and passes from infinity to zero. The fraction $(r-g)/(g-v)$, which may be called the "Colour Index," has therefore in the spectrum every value from plus to minus infinity, and has all the positive values twice over. Every positive colour index has two spectrum colours:—one in which the order of magnitude of the sensations is Red, Green, Violet, and the other in which the order is Violet, Green, Red. In fig. 1, where the order is that required, let the lines S, S′ give the spectrum colours. Then it is clear that these two spectrum colours are complementary to one another. Also that the colour represented by A is equal to the spectrum colour S plus the colour included between S and A, which is white; and also that the colour at A plus the spectrum colour at S′ form the white between A and S′.

Now suppose the colour index negative, then R, G, V must be arranged in the order R V G or V R G (see fig. 3). We have A, A′, and S, as before; but S′, the second spectrum colour, does not occur, inasmuch as there is no spectrum colour in which green is less than both the red and the violet. Hence the green, which is represented by A, can be defined only by the addition of white to a spectrum colour; and the purple, which is represented by A′, can be defined only by

the fact that when added to a spectrum colour they can form white.

To see how what precedes is represented in the Colour Map (Plate XI. fig. 4), take any line perpendicular to the spectrum-line, say the line in the orange for which the Colour Index is 1·0, and compare this line with fig. 1. S is the point on the spectrum-line, V is the point at which the "line of no violet" is crossed, and G and R the points in which the lines of no green and no red respectively are crossed, and S′ represents the complementary spectrum colour, which is represented on the thick line at the point marked 1·0. This thick line, along which the figures are marked, represents the spectrum which is complementary to that from Red to Yellow, and itself extends from Violet, of which the colour index is zero, to Blue, of which the colour index is infinite. A similar line gives the complementary spectrum of the part from Blue to Violet, and itself extends from Red when the colour index is infinite to Yellow when it is zero. The region on the right outside *all* the lines gives all the colours to be obtained by adding white to a spectrum colour; and to ascertain the amount of each sensation, we have only to measure horizontally to the line giving the zero of that sensation. The region on the left outside *all* the lines gives all the colours capable of making white with spectrum colours; and here, again, to ascertain the amount of each sensation we have only to measure horizontally to the line giving the zero of that sensation. It will thus be seen that the whole map is really constructed on one single principle. It is obvious that if a series of colours are obtained by some definite law, their positions on the map will lie on some line straight or curved.

It remains to consider the spaces enclosed within the lines. On the right between the spectrum-line and the nearest sensation zero-lines lies a space which has a real meaning, as the points in it represent colours in which the sensations have certain positive ratios to one another; but these ratios give a more intense colouring than the spectrum colours themselves, and therefore such points cannot represent any colours which can be seen by a normal eye, because, as was known to Newton, every mixture of colours is more diluted than the spectrum colour which it most nearly resembles. This region may be

called an abnormal region. The colours it represents would be visible to eyes more or less colour-blind. There are two abnormal regions on the left of the figure between the complementary spectrum-lines and the red and violet zero-lines respectively.

The remaining portion of the map, viz. that lying *between* zero sensation lines, is of a different nature. At any point in this region the distances measured to the zero lines are not all in the same direction; so that one or two out of the three sensations must be considered to be negative. As no one possesses a negative colour sensation, the colours represented in this region are imaginary. This may be called the imaginary region. Though it has no physical meaning it will be found to have its value in connexion with the geometrical structure of the map. As an example of this, consider the complementary spectrum-lines. They end abruptly, leaving a gap opposite the green; but they may be continued across the gap in such a way as their general form seems to point, and this has been done in fig. 4, by continuing the complementary spectrum-lines until they meet in a cusp at the point on the right marked $-1{\cdot}0$. This extension lies wholly in the imaginary and abnormal region, and may represent the missing complementary spectrum of green.

The map affords convenient methods for calculating the effect of mixing colours. Let a colour which has the sensation red, green, and violet in the proportion r_1, g_1, v_1 be represented by $r_1 \mid g_1 \mid v_1$. Then, if we take two colours $r_1 \mid g_1 \mid v_1$ and $r_2 \mid g_2 \mid v_2 \mid$, the mixture of these colours in the proportions l_1 and l_2 will give the result $l_1r_1+l_2r_2 \mid l_1g_1+l_2g_2 \mid l_1v_1+l_2v_2$. The index of this colour is

$$\frac{l_1(r_1-g_1)+l_2(r_2-g_2)}{l_1(g_1-v_1)+l_2(g_2-v_2)}.$$

Let the spectrum colour having the same index be $r \mid g \mid v$. In order to find the quantity of white which must be added to this spectrum colour to produce the required colour, it is necessary that the luminosity of the colour should be altered to that luminosity at which the colour is represented in the map. This can be done by multiplying the coefficient of each sensation by the fraction $(r-g)/\{l_1(r_1-g_1)+l_2(r_2-g_2)\}$ or one

of the equivalent fractions. The resulting sensation coefficients are

Red . . . $(l_1r_1+l_2r_2)(g-v)/\{l_1(g_1-v_1)+l_2(g_2-v_2)\}$.
Green . . $(l_1g_1+l_2g_2)(v-r)/\{l_1(v_1-r_1)+l_2(v_2-r_2)\}$.
Violet . . $(l_1v_1+l_2v_2)(r-g)/\{l_1(r_1-g_1)+l_2(r_2-g_2)\}$.

The coefficient of the white to be added to the spectrum colour is obtained while the colour lies on the right hand by subtracting r from the red coefficient above obtained, or by subtracting g and v from the other coefficients respectively. When the colour lies on the left hand, the white is obtained by adding r to the above red coefficient, or g and v to the green and violet coefficients respectively.

I have applied the formulæ given above to obtain the curves showing the results of adding together in any proportion two spectrum colours so related to each other that if the first is $r \mid g \mid v$ the second is $v \mid g \mid r$. The index of the second colour being $\frac{v-g}{g-r}$ is the reciprocal of the index of the first.

The curves obtained are shown in fig. 5. Consider the curve numbered 2. This is the locus of mixtures of the blue whose index is 2·0, and the yellow whose index is 0·5. The curve passes through these two points of the spectrum, giving the cases in which a zero quantity of one of the colours is taken; and every other mixture is indicated by some point on the curve joining these two points and lying to the right of the spectrum-line. In this figure the dotted horizontal lines occupy the positions where the indices are zero and infinity respectively, so that the portion of any curve which lies outside of them must be repeated again on the left side of the complementary spectrum-line. In curve No. 2 two small parts do lie outside the dotted lines, and, accordingly, these two parts are repeated to the left of the complementary spectrum-line. We have then the curve No. 2 in three separate portions, which it is not possible to connect physically, as the missing part of the curve lies in the imaginary and abnormal regions. But what is not possible for physics is easy for geometry. We cannot subtract one spectrum colour from another, but we can subtract the lines representing the sensations in one

spectrum colour from the lines representing the sensations in the other spectrum colour ; and so by subtracting one spectrum colour from the other in any proportions we can complete the curve No. 2 through the imaginary and abnormal regions and so obtain the complete and continuous curve. Curves Nos. 1 and 0 have no portion on the complementary side, but curves Nos. 3 and 4 have a considerable portion on that side. A new feature is shown when we take the locus numbered 5. This is got by combining the spectrum indigo, having index 1·0, with spectrum orange, having the same index. These are complementary colours. When added together in the proper proportion they produce white, and when added in any other proportion they produce white plus whichever spectrum colour predominates. Hence the locus consists of horizontal straight lines through the two points in the spectrum-line, going off to infinity, where the colour indicated is white, considered as a spectrum colour infinitely diluted with white light. Next consider curve No. 7. The main portion of the curve lies to the left, and starts from points in the complementary spectrum-line which indicate the spectrum colour chosen. The parts of this portion which lie outside the horizontal dotted lines are repeated to the right of the spectrum-line; the remainder, obtained by subtraction, lies wholly in the imaginary and abnormal regions. All these curves pass through a certain pair of points, as may be easily shown.

The first spectrum colour is . . . $r \mid g \mid v$.
The second is . . . . $v \mid g \mid r$.

By subtraction of one from the other, we get a colour

$$r-v \mid \text{zero} \mid v-r.$$

The resulting colour has therefore no green, and has the red and violet equal in amount but opposite in sign. These conditions are satisfied at the two points shown in the figure.

In this figure the lines are drawn under the condition that the index of one spectrum colour is the reciprocal of the index of the other ; but any number of other systems of lines might be drawn showing combinations of two spectrum colours, so that it is evident that every colour can be resolved into two spectrum colours in an infinite number of ways.

There are three regions in fig. 5 which are shaded to show

that none of the curves pass through them. These regions might probably be filled up by curves drawn through points in the imaginary part of the complementary spectrum to which I have already alluded.

Now the complementary spectrum-line and the curves giving mixtures of two spectrum colours have been drawn by strict arithmetical methods from certain curves of hypothetical form which indicate the intensity of the sensations for each point of the spectrum; but they can also be plotted out by direct experiment.

To plot out the complementary spectrum-line, add to a spectrum colour its complementary until white is produced, measure the quantity of white, and mark off a horizontal line to the *left* from the point in the spectrum of a length proportional to the quantity of white. The end of this line is a point in the complementary spectrum; other points may be obtained in the same way, and the normal part of the complementary spectrum-line be drawn.

To plot out the curve giving the mixtures of two spectrum colours, take a third spectrum colour and make a colour patch of the first two colours, and another colour patch of the third colour and white. Keep the luminosity of the third colour constant, and vary that of the other colours and the white until both patches are of the same colour. Then measure the quantity of white used and mark off a line from the position of the third spectrum colour to the right proportional to the quantity of white. The end of this line gives a point in the curve. By taking other spectrum colours as the third colour other points may be obtained. If, however, it is found impossible to make the two patches of the same colour, then throw the three spectrum colours together, and keeping the luminosity of the third colour constant vary that of the other two until the three produce white; measure the quantity of white, and mark off to the left from the position of the third spectrum colour a line proportional to the quantity of white obtained. The end of this line is a point in the curve. If both these methods fail the point on the curve corresponding to the third spectrum colour lies in the abnormal or imaginary regions, and cannot be determined by experiment.

When the derived curves have been plotted out by experi-

ment, it will be possible to modify the hypothetical forms of the curves of intensity of the sensation in the spectrum so as to make the curves derived from them accord more closely with the results of experiment, and so to arrive by gradual approximation to the true form of those curves.

XXXVI. *A Note on the Electromotive Forces of Gold and of Platinum Cells. By* E. F. Herroun, *Professor of Natural Philosophy in Queen's College, London* *.

In nearly all modern text-books of Physics the metal platinum is placed after gold in Volta's Electropositive Series. This no doubt is partly owing to the well-known fact that gold is attacked by chlorine or nitrohydrochloric acid more readily than platinum, and it might therefore be reasonably supposed that gold evolves more heat in the formation of its chloride than does platinum. On referring to the values for the heats of formation of the chlorides of these two metals, as given by Julius Thomsen †, one finds, however, that the heat attending the formation of auric chloride is, per equivalent, only about half as great as that in the case of platinic chloride.

Assuming that the voltaic constants of metals are deducible from the thermochemical values of their compounds, the above facts would compel us to regard gold as more negative than platinum, at least when immersed in chloride solutions. (The same observations would also apply if oxygen were the attacking medium, as Thomsen gives the heat of formation of platinic hydrate as a considerable positive number, while that of auric hydrate is a large *negative* quantity.)

It was, therefore, an interesting point to determine how far the actual electromotive forces obtained with gold and with platinum agreed with these conclusions, and I endeavoured to find records of the electromotive forces of cells in which these metals are immersed in solutions of their chlorides opposed

* Read March 25, 1892.

† *Thermochemische Untersuchungen*, iii. pp. 412 & 430.

to some other metal in a solution of its corresponding salt. While there are many references to the E.M.F.'s set up in single fluid cells in which platinum is one of the metals used, the references to gold are scanty, and even with platinum I have only succeeded in finding one recorded measurement in which the platinum was immersed in a solution of its own salt. This was a measurement made by Wheatstone *, in which liquid zinc amalgam was opposed to platinum in a solution of platinic chloride. He found in measuring the E.M.F. of this cell that it required 40 turns of his rheostat, as compared with 30 turns required for a form of Daniell cell. Now, assuming his Daniell cell to have had an E.M.F. of 1·09 volt, the value in volts for the zinc amalgam, platinum-platinic chloride cell would be 1·453.

In a list of the potential differences between different metals and graphite simply immersed in water, Götz and Kurz † give the values 0·48 volt for gold and 0·37 volt for platinum, the value for zinc and graphite being 1·37 volt. This would make platinum more electronegative than gold by 0·11 volt: but these values cannot be accepted as in any degree expressing the actual electromotive forces concerned.

Exner and Tuma ‡, on the other hand, taking carbon $=0$, give $\mathrm{Pt} = 0{\cdot}05$, $\mathrm{Au} = -0{\cdot}05$, which appear to be much more probable values, and make gold, as its thermochemical data require, negative as regards platinum.

Under these circumstances it appeared worth while measuring the actual E.M.F. set up between zinc and gold, and zinc and platinum, in solutions of their own chlorides of equal molecular strength.

Zinc-Platinum Cell.

The heat of formation of $PtCl_4$ is apparently unknown, but as it appears to be impossible to prepare a *neutral* solution of that salt, compounds such as $PtCl_4,2HCl$ or $PtCl_4,2NaCl$ must be substituted, and their heats of formation are given by Thomsen. I selected the latter salt on account of its freedom from acid, and prepared a neutral solution having

* Wheatstone's Scientific Papers, p. 115.

† *Electrotechnie Zeit.* ii. p. 30.

‡ *Wien. Ber.* xcvii. p. 917.

the strength of $\cdot25(PtCl_4 2NaCl)100H_2O$, which therefore contained about 2·75 grams of Pt in 100 cub. centim. of solution.

Thomsen gives for [$Pt, Cl_4, 2NaCl$, aq] the value 73720 + 8540 = 82,260 calories; and for [Zn, Cl_2, aq] the value 112,840 calories. These numbers would give as the heat of replacement of one equivalent of platinum by zinc the nett heat evolution of 35,855, which is equivalent to a theoretical E.M.F. of 1·548 volt.

A cell was set up consisting of an amalgamated pure zinc rod immersed in a solution of $\cdot25\ ZnCl_2 100H_2O$, opposed to a clean platinum plate immersed in the solution of sodio-platinic chloride above described. The two solutions were separated both in this and other experiments by an ordinary porous earthenware pot, and the E.M.F. was measured by balancing it against a difference of potential by Poggendorff's method.

The standards taken were a Latimer-Clark cell, which was assumed to have an E.M.F. of 1·435 volt at 15° C., and a chloride of silver battery (modified De-la-Rue cell), which by comparison with the Clark cell was found to have an E.M.F. of 1·045 volt. I find this cell more convenient in using Poggendorff's, or any similar method, as its E.M.F is not appreciably disturbed by its sending a small current, or by shaking, and it has a smaller temperature-coefficient than the Clark cell. The temperature of all the cells used in these experiments only varied between the narrow limits of from 12° to 15° C.

In one experiment the zinc-platinum cell, when first set up, gave an E.M.F. of 1·647 volt. It was then allowed to send a current through a low external resistance for five minutes, and after further resting for five minutes its E.M.F. was again measured, when it was found to have dropped to 1·473 volt. After a further rest of about ten minutes it recovered to 1·507 volt, at which value it remained tolerably constant. During the passage of the current from the cell the platinum plate became covered with a black deposit of finely divided platinum, and I thought it not improbable that this alteration of the surface might be the cause of the marked falling off in E.M.F.; but on removing the solution surrounding the platinum plate and replacing it with fresh, the E.M.F. regained its initial high value although the surface of the plate had not been

disturbed. It appeared from this that the high initial value was probably due to oxygen dissolved in the liquid, which the platinum would be very apt to occlude superficially, and which would account for the uncertain values of the E.M.F.

Whatever may be the cause, this variability renders the measurement of the E.M.F. of this form of cell very difficult.

In other experiments values as high as 1·7 volt were obtained on first setting up the cell, which, after sending a current and resting, fell to the tolerably stable value of about 1·525..

Maximum	E.M.F.	= 1·70	volt.
Minimum	„	= 1·473	„
Average	„	1·525	„

The average value (1·525) is seen to be slightly lower than the E.M.F. calculated from the thermochemical equation (1·548); but the difference is small (·023 volt), and is well within the limits of experimental error with such a variable cell. There is therefore no reason to assume that its actual E.M.F. departs from the theoretical value.

Zinc-Gold Cell.

The heat of formation of [Au,Cl_3,aq] is given by Thomsen as 27,270 calories, and that of [Zn,Cl_2,aq] being 112,840, the difference per equivalent gives 2·044 volts as the theoretical E.M.F. of zinc, displacing gold from weak solutions of its chloride.

A cell consisting of an amalgamated zinc rod immersed in a solution of zinc chloride having the strength of $\cdot 25ZnCl_2 100H_2O$ opposed to a plate of gold in a solution of auric chloride of equal molecular strength, was set up and its E.M.F. immediately measured. It was found to give an E.M.F. of 1·855 volt, and after actual short-circuiting for five minutes it had only fallen to 1·834. These values were in fact the extreme limits of the variations that I have observed on repeating the experiment, and the constancy of this cell contrasts in a striking manner with the variability of the platinum cells.

Maximum E.M.F. = 1·855 volt.

Minimum „ = 1·834 „

Mean „ = 1·844 „

Assuming the thermal values for gold to be accurate, the actual E.M.F. thus measured is seen to be ·2 volt below the calculated value; or, adopting the convention suggested by Messrs. Wright and Thompson, −0·2 volt is the *thermo-voltaic constant* for gold in dilute neutral solutions of its chloride.

When a platinum plate was substituted for the gold plate and immersed in the gold-chloride solution, the E.M.F. of the cell thus formed was found to be 1·782 volt, *i. e.* less than the gold-gold chloride, but greater than the platinum-platinic chloride cell given above.

From the thermochemical values one might conclude that platinum would be capable of replacing gold from gold chloride; but, so far as my experiments have gone, I have not found this to be the case, nor on the other hand is gold able to replace platinum from platinic chloride, which, of course, is not to be expected.

When a plate of gold and another of platinum are immersed together in pure water or in dilute hydrochloric acid, the gold acquires slightly the higher potential. If strong hydrochloric acid be substituted for the dilute, the direction of the difference of potential becomes doubtful, and on the addition of nitric acid to the strong hydrochloric, so as to form *aqua regia*, the platinum acquires distinctly the higher potential, and if the outside circuit be closed on a galvanometer, a very decided current flows from the gold to the platinum through the cell (*cf.* Ganot's Physics, article on "Electromotive Series").

Why the nascent chlorine combines readily with the gold where it evolves but little heat and slowly and reluctantly with the platinum, in which reaction much more heat is disengaged, is a problem which at present appears to admit of no satisfactory solution.

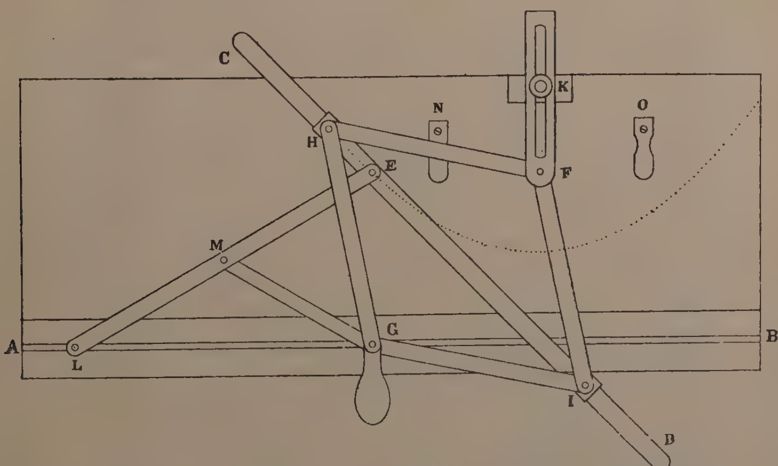
XXXVII. *On an Instrument for Drawing Parabolic Curves.* *By* Richard Inwards, *F.R.A.S.**

This instrument, which I have now the pleasure of showing to the members of the Physical Society, is designed for the purpose of drawing by one simple operation any parabola of short focus, such as might be wanted for setting out the curve of a lamp-reflector, or in making a diagram of the path of a comet or of a projectile.

It is based on that property of the parabola by which any point in it is equally distant from its focus and from the nearest part of its directrix.

In the diagram, which represents the instrument as mounted on a drawing-board, F is the focal pivot, which is adjustable for larger or smaller parabolas by moving the piece supporting it in a slide to which it can be clamped by the screw K.

AB is a slot formed by two steel straight edges with a



narrow opening between them. The centre line of this slit is the directrix of the parabola to be drawn.

Along the slit AB travel two pins L and G, which are parts of a frame or system LMEG of such a nature that LM, ME, and MG are equal to each other, so that in any position E is compelled to be vertically over G.

A parallel frame HGFI is so constructed that one corner

* Read May 13. 1892.

rotates on F and the opposite corner rotates on G. The diagonal bar CD can travel along between guides at H and I.

A pencil or scriber is fixed at E, and the paper is placed under this, and held by the clips N and O.

A handle is provided at G, on moving which along the line AB it will be seen that the pencil E is compelled to describe a curve such that it is always equally distant from F and from G, and as G is on a straight line and F is a fixed point, that curve must be a parabola.

The fact that any point in the diagonal of a rhombus must be equally distant from one pair of its opposite angles is obvious on a moment's consideration.

The centre line of the diagonal bar CD is always a tangent to the parabola which is being constructed. The curves so drawn may be used for the production of templates for lenses or mirrors, and they could be drawn small and then magnified either by photography or by a pantagraph arrangement.

This instrument is a combination of well known link movements, but I do not think they have ever before been applied to the production of a parabolic curve from a single straight line motion.

The instrument may be so constructed that any play between the sliding pins and the slot may be avoided by pressing the handle down towards the lower side of the slot, which thus becomes a ruler. In the event of any machine being constructed on this principle, gravity itself might make this pressure.

20 Bartholomew Villas, Kentish Town, N.W.,
February 2, 1892.

XXXVIII. *A Portable Instrument for Measuring Magnetic Fields. With some Observations on the Strength of the Stray Fields of Dynamos.* *By* EDWIN EDSER *and* HERBERT STANSFIELD*.

[Plates XII. & XIII.]

THIS instrument was constructed for the purpose of giving direct readings for the strength of magnetic fields, such as are found in the neighbourhood of Dynamos; thus avoiding

* Read May 13, 1892.

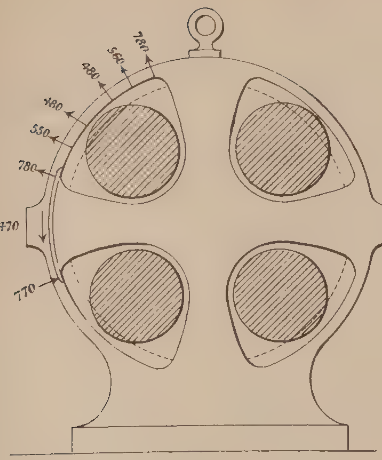


Fig. 6.

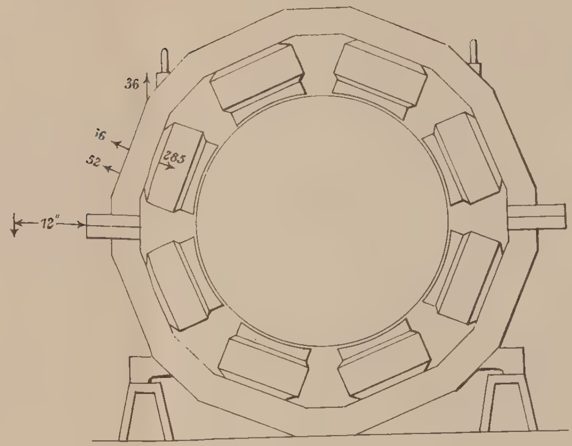


Fig. 4.

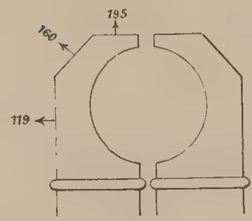


Fig. 7.

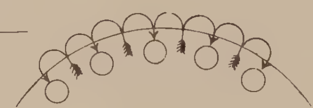


Fig. 5(b).

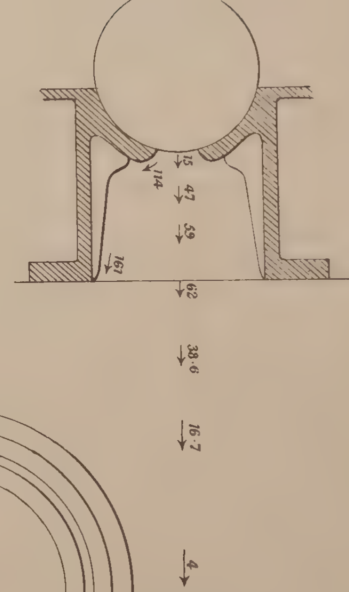


Fig. 5(c).

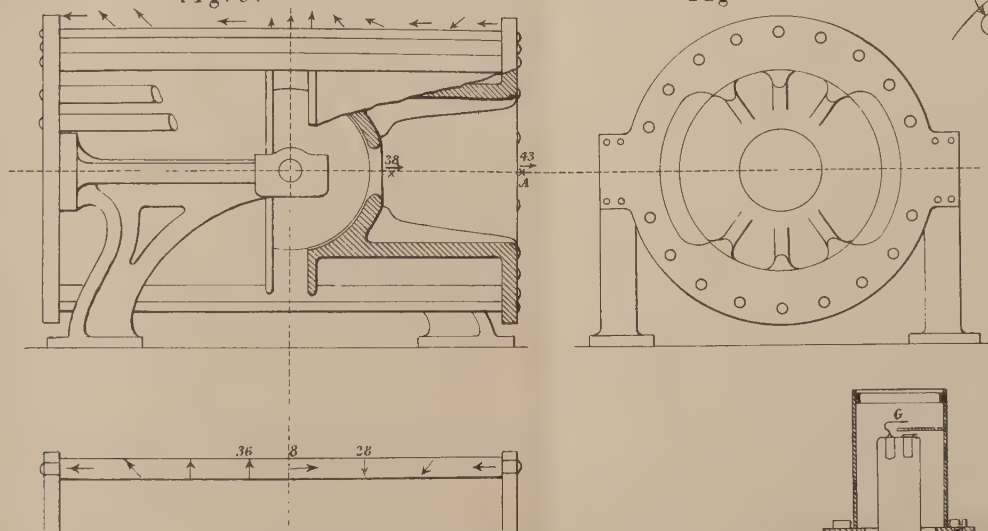


Fig. 5. (a).

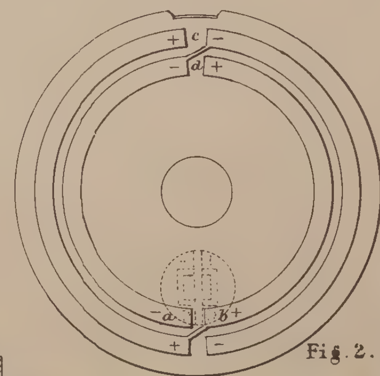


Fig. 2.

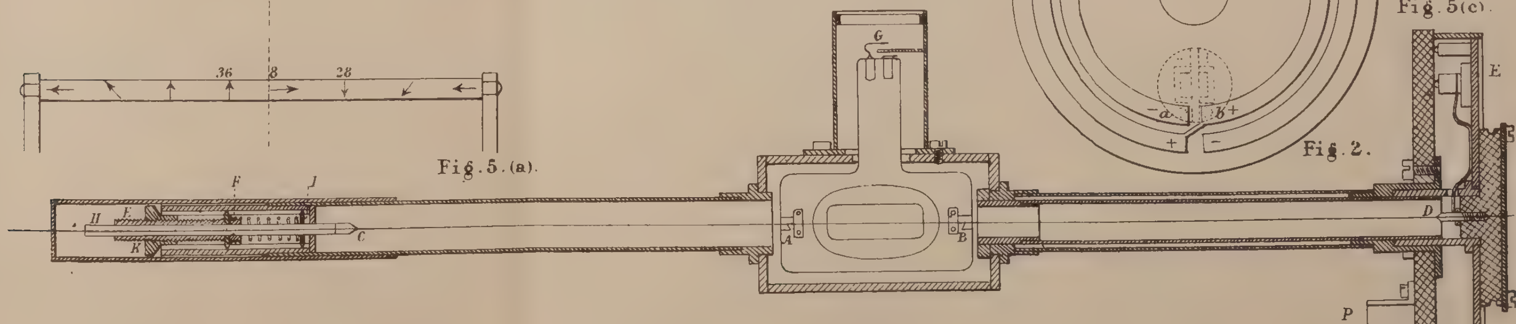


Fig. 1.

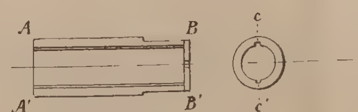
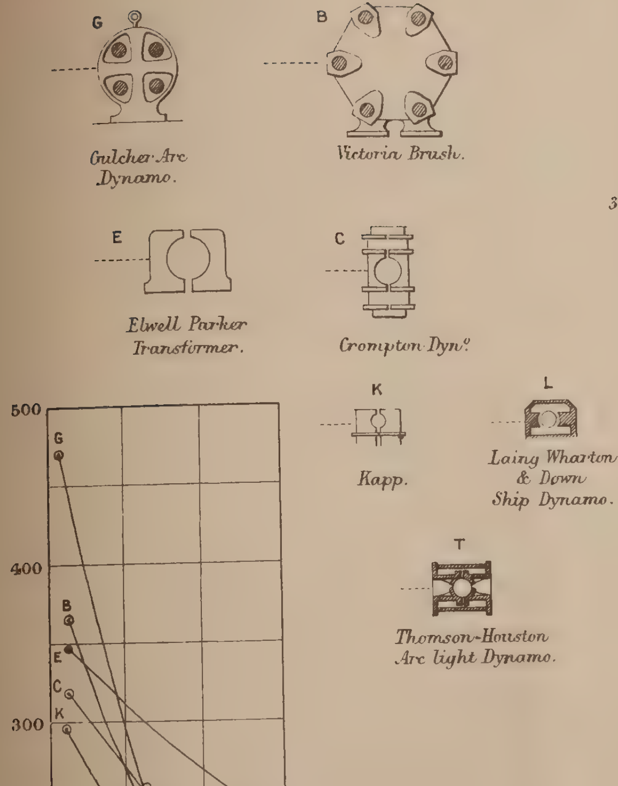


Fig. 3.



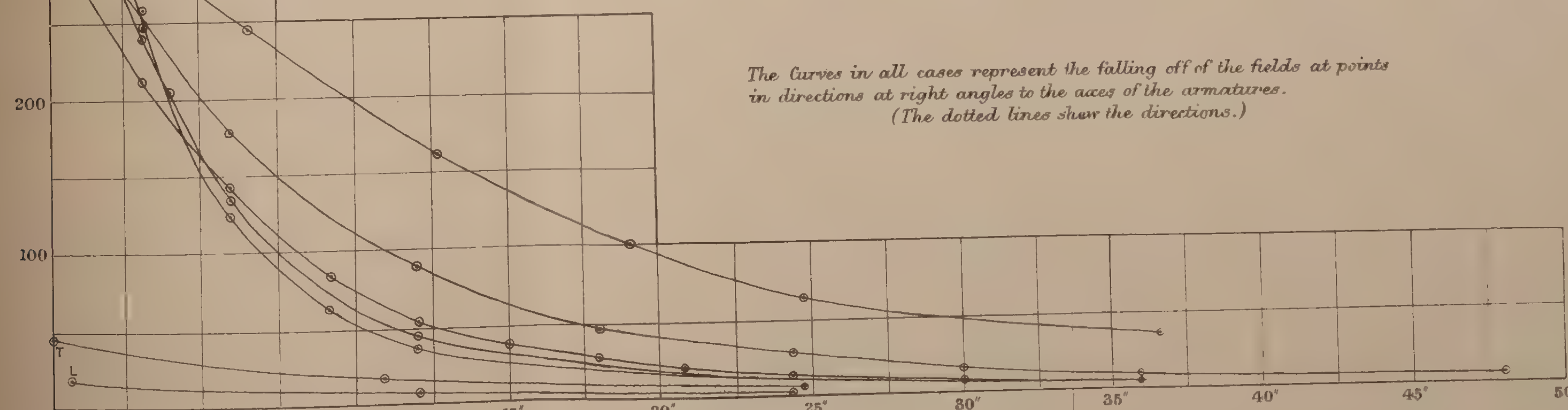


Fig. 1.

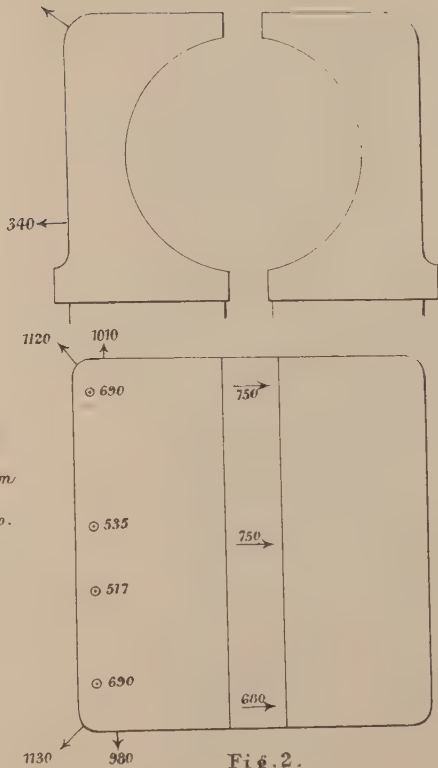


Fig. 2.

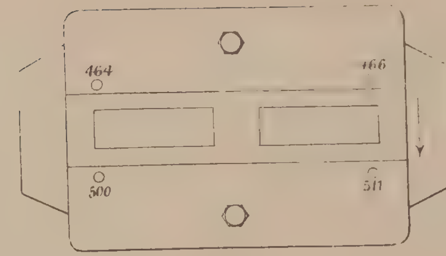


Fig. 3.

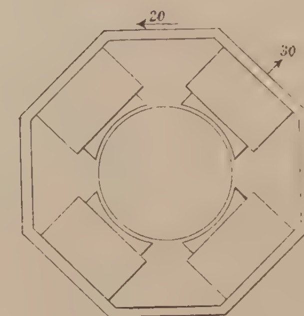


Fig. 4.

The Curves in all cases represent the falling off of the fields at points in directions at right angles to the axes of the armatures.
(The dotted lines shew the directions.)

the inconveniences attending the Ballistic method. Portability, a considerable range, and a fair degree of accuracy were the qualities sought. The instrument, as now constructed, whilst satisfying the first of these conditions, may be used to measure any field from 1 line per centimetre upwards, with an error of only about 2 per cent.; the only accessories required being a dry cell and a resistance-box.

In principle it is the inversion of the D'Arsonval galvanometer; the torsion necessary to restore a coil, through which a constant current circulates, to its normal position, parallel to the direction of the lines of force, furnishing readings proportional to the field at the position of the coil*.

A diagram of the instrument is shown in fig. 1 (Plate XII.). A B is a small coil, oblong in shape, wound of No. 44 B.W.G. copper wire, and suppported half on each side of a sheet of mica. It is suspended from each end by strips, 10 centim. long, of rolled German-silver wire, each strip having a loop which is passed over a small brass hook riveted on to the mica, and in electrical communication with a terminal of the coil. The strip C A is in electrical connexion with the case of the instrument at C, whilst the strip D B is insulated from it at D by an ebonite plug, attached to the torsion-head E. Inside this latter is a commutator for automatically reversing the current, so as to take readings on each side of the zero. It consists of four semicircular strips of copper, cross connected as shown in fig. 2, *a* and *b* being connected to the two battery terminals. Two springs, one soldered to the case, the other insulated from it, but connected to the end of the suspending strip D B, press on these semicircular strips. When the torsion-head is at zero no current passes, the springs then

* After the completion of this instrument our attention was called to some experiments of Messrs. Siemens and Halske, in which the same principle was used.

"In order to measure the intensities of the rotary field, a coil was hung in the centre of the ring, in such a way that its magnetic axis was perpendicular to the measured direction of the resulting magnetic axis of the ring. The coil was then excited by a continuous current, and was kept in position by a spring. The torque of the spring served as a measure of the intensity." "Deduction and Experiments on Rotary Currents." A. du Bois Reymond, 'Electrical Review,' June 5, 1891, vol. xxviii. No. 706.

being at *c* and *d* respectively (fig. 2). To take a reading the torsion-head is turned, thus sending the current through the coil. Should the latter be deflected in the wrong direction, the current can be reversed by means of the plug contact, P, attached to the battery leads. Readings are taken on each side of zero in order to eliminate any error due to imperfect balancing of the coil; an aluminium pointer G, attached to the coil, being always brought by the torsion to the zero position on a small scale.

In order to obtain at once a spring-suspension and an adjustment for the torsion of the strip, a particular form of geometrical slide is used. A A′ (fig. 3) is a thick brass tube, turned at B B′ to a slightly conical plug to fit the tube of the instrument (fig. 1). Two grooves (seen in plan at C C′) are made along this tube, a cross head F (fig. 1) on the screw E F fitting into them. This screw is drilled along its whole length to admit a thick wire H I C, the latter having a cross head I, also fitting into the longitudinal grooves. These two cross heads are then connected to the two ends of a spiral spring, in such a manner that they are pressed by it against opposite sides of the grooves. The suspending strip being connected to the central wire H I C at C, its tension can be increased or diminished by means of the nut K, without altering the position of the coil. Any sudden jerk will also be taken by the spring, thus obviating the risk of stretching the suspending strip. Scratches on the wire H I C near H indicate the tension used.

As a source of current a Hellesen dry cell is used. When joined up through 50 ohms the E.M.F. of the cell is practically constant, whilst its internal resistance is negligible*.

The resistance of the instrument having been made up to 50 ohms, it follows that its sensitiveness can be varied by introducing an independent resistance in the circuit.

Let C = constant of instrument (*i. e.* field for 1° of torsion, with no external resistance in circuit);

* See *Electrotechnische Zeitschrift*, August 1, 1890, vol. ii. No. 31. Republished in pamphlet form by Siemens Bros. and Co., Ltd. We have independently verified these results.

n = multiple of 50 ohms in circuit, exclusive of resistance of instrument;

θ = mean angular torsion;

then

Field in C.G.S. measure $= C(n+1)\theta$.

C was determined, and the instrument calibrated, between the coils of a galvanometer of the Gaugain type through which a known current was passed. For an E.M.F. of 1·45 in the dry cell it was found to be ·293. The error shown in the calibration was always below 2 per cent.

By permission of the Committee of Experts, and of a number of the firms exhibiting, a series of measurements were made at the Electrical Exhibition, Crystal Palace. The results obtained are shown in the remaining figures.

Fig. 1 (Plate XIII.) shows the fields measured at various distances from different dynamos, the distances plotted as abscissæ and the fields as ordinates. It is noticeable that machines of the multipolar type show a much steeper curve than other dynamos. This is especially noticeable in the case of the Gulcher Dynamo curve (G).

Fig. 4 (Plate XII.) shows the fields round Mr. Kapp's 8-pole machine. They are noticeably small.

Fig. 2 (Plate XIII.) shows the effect of edges, corners, &c. on the strength of field. On the flat surface of the pole-piece the field was about 600 (varying between 517 and 690), on the edges increased to about 1000, whilst on the corners it reached a strength of over 1100 C.G.S. lines per centimetre.

Fig. 3 (Plate XIII.) shows the deformation of the stray field produced by the armature reactions. The measurements were made on an Elwell-Parker Motor. The strength of field on the trailing edge was about 460, whilst that on the leading edge was about 500.

Fig. 6 (Plate XII.) shows various measurements made on the Gulcher Dynamo; Fig. 7 (same Plate) gives the field near one of Mr. Kapp's Dynamos; Fig. 4 (Plate XIII.) shows the fields at two positions of a Laing, Wharton, and Down shielded dynamo.

Some curious effects of armature reactions are noticeable on the Thomson-Houston Dynamo (fig. 5, Plate XII.). As the bars in this machine act as a yoke, the result is due to combined magnetic leakage and armature reaction.

By the kindness of Mr. Harrison we were enabled to make some experiments on a watch, previously unmagnetized, which he lent us. We found that a field of about 10 had no appreciable effect on its rate of going, but that after being subjected to a field of about 40 it lost about 8 minutes per day; and even after being demagnetized in an alternating field it still continued very erratic in its actions. Of the dynamos whose fields we have measured, with the exception of the Thomson-Houston, Ship's Dynamo (Laing, Wharton, and Down), and Mr. Kapp's large Multipolar, it would not be safe to go nearer than about 2 feet*. Moreover, with a watch with a steel balance-wheel (the one experimented upon had a brass one), even greater precautions might have to be observed.

Finally, we wish to record our thanks to Mr. Harrison for allowing us to experiment on his watch; to Mr. Barton for his assistance in constructing the instrument; and to Messrs. Crompton, Kapp, Laing, Wharton, and Down, the Gulcher Company, the Electric Construction Corporation, and Major-General Festing, for permission to experiment on their various dynamos, and also to publish the results.

XXXIX. *Note on the Measurement of the Internal Resistance of Cells.* *By* E. Wythe Smith†.

In order to determine the actions which take place in an accumulator during charge and discharge, it is necessary to know the working electromotive force at the different stages. This might be observed by breaking the circuit; but immediately on doing this the electromotive force varies at a very rapid rate, so that if only four or five seconds be occupied in taking the measurement an error of 25 per cent. may be made in the difference between the electromotive force and the terminal potential difference. If time-readings be taken after breaking the circuit and a curve drawn connecting E.M.F. and time, this curve may be produced back

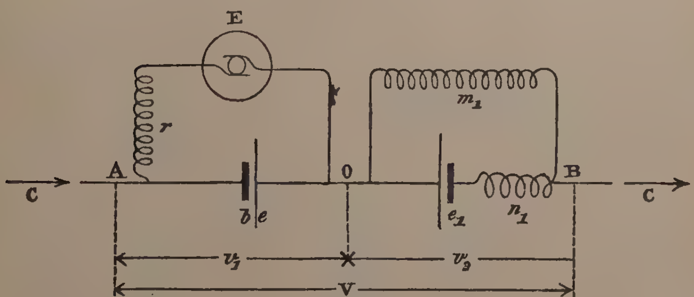
* One could not safely go within three feet of the Elwell-Parker Continuous Current Transformer.

† Read June 24, 1892.

in the way described by Prof. Ayrton and others in a paper at the Institute of Electrical Engineers. But, as this method has its objections in addition to that of interrupting the circuit, it is very desirable to determine the actual working E.M.F. in some other way.

The E.M.F. could be readily obtained from the terminal P.D. and the current if the internal resistance were known. It is for the determination of this latter quantity that I have devised the following modification of Mance's method. Of course this method is applicable to the measurement of the resistance of other forms of battery besides accumulators.

Fig. 1.



In fig. 1 let the cell of E.M.F. e, and internal resistance, b, be the one experimented upon, r being the resistance of the external circuit which may contain an E.M.F. E, for example that of the dynamo used to charge the cell. This circuit is connected at O with an auxiliary circuit, in which the resistances m_1 and n_1 are so adjusted that the points A and B are at the same potential, the resistance of the cell of E.M.F., e_1, being included in n_1. Suppose a current, C, from some external source to pass through both circuits in series. The P.D. between A and B will now be V. Let P.D. between A and O be v_1, and that between O and B be v_2, $(v_1+v_2)=\mathrm{V}$. We have

$$\mathrm{C}=\frac{v_1+e}{b}+\frac{v_1+\mathrm{E}}{r}=\frac{v_2-e_1}{n_1}+\frac{v_2}{m_1};$$

therefore

$$v_1=\frac{br\mathrm{C}-re-b\mathrm{E}}{b+r},$$

and

$$v_2=\frac{m_1n_1C+m_1e_1}{m_1+n_1},$$

$$V=\frac{b+C-re-bE}{b+r}+\frac{m_1n_1C+m_1e_1}{m_1+n_1}.$$

But when $C=0$, $V=0$,

$$\therefore\ 0=\frac{-re-bE}{b+r}+\frac{m_1e_1}{m_1+n_1};$$

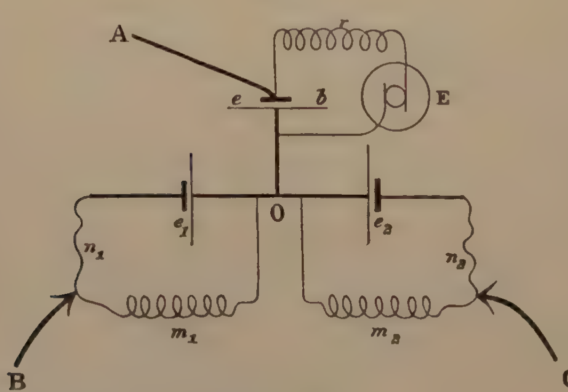
$$\therefore\ V=C\left(\frac{br}{b+r}+\frac{m_1n_1}{m_1+n_1}\right).$$

If we measure the apparent distance R_1 between A and B by any convenient method, and if C be the current sent through this circuit between A and B by the testing arrangement, we get R_1 equal to $\frac{V}{C}$;

$$\therefore\ R_1=\frac{br}{b+r}+\frac{m_1n_1}{m_1+n_1}.$$

Now if we have three circuits (fig. 2) connected together

Fig. 2.



at the point O, the cell of E.M.F. equal to e and resistance b being the particular one whose resistance is required, and if the resistances m_1, m_2, n_1, n_2 be so adjusted that the points A, B, and C are at the same potential, the apparent resist-

ances R_1, R_2, R_3 between the points A and B, A and C, and B and C, will have the following values :—

$$R_1 = \frac{br}{b+r} + \frac{m_1 n_1}{m_1+n_1},$$

$$R_2 = \frac{br}{b+r} + \frac{m_2 n_2}{m_2+n_2},$$

$$R_3 = \frac{m_1 n_1}{m_1+n_1} + \frac{m_2 n_2}{m_2+n_2};$$

$$\therefore \frac{br}{b+r} = \frac{R_1+R_2-R_3}{2}, \text{ say, equals } x,$$

then the required resistance of the cell,

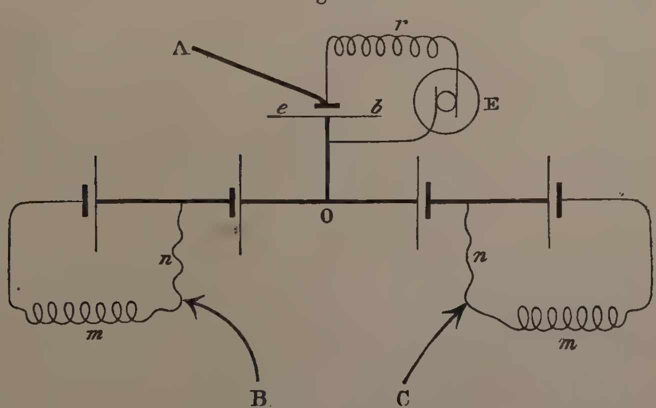
$$b = x + \frac{x^2}{r} + \frac{x^3}{r^2} + \&c.$$

If, as in the case of an accumulator, x is small compared with r, then

$$b = x + \frac{x^2}{r}.$$

When an accumulator is discharging, taking $b=x$ gives us a a value for b about 2 per cent. too low.

Fig. 2 *a*.



If the P.D. at terminals of the cell under test is greater than the E.M.F. of a single balancing cell (as is the case

during charge) then the circuit must be modified as shown (fig. 2 *a*).

If a Wheatstone's bridge be employed to measure R_1, R_2, and R_3, there will be no necessity to employ any special instrument in the testing for equality of potential of the points A, B, and C. For all that need be done is to remove an infinity plug on the bridge, close the galvanometer circuit, but not that of the testing battery, and adjust m_1, m_2, n_1, n_2 until the galvanometer remains at zero.

XL. *Breath Figures.*
By W. B. Croft, *M.A.**

Fifty years back Prof. Karsten, of Berlin, placed a coin upon glass, and by electrifying it made a latent impression, which revealed itself when breathed upon. About the same time Mr. W. R. (now Sir W. R.) Grove made similar impressions with simple paper devices, and fixed them so as to be always visible. A discussion of Karsten's results occurs in several places, but I have not been able to find details of his method of performing the experiment. During my attempts to repeat it some effects have appeared which seem to be new and worthy of record.

After many trials I found the following method the most successful:—A glass plate, 6 inches square, is put on the table for insulation : in the middle lies a coin with a strip of tinfoil going from it to the edge of the glass : on this coin lies the glass to be impressed, 4 or 5 inches square, and above it a second coin. It is essential to polish the glass scrupulously clean and dry with a leather : the coins may be used just as they usually are, or chemically cleansed, it makes no difference. The tinfoil and the upper coin are connected to the poles of a Wimshurst machine which gives 3 or 4 inch sparks. The handle is turned for two minutes, during which one-inch sparks must be kept passing at the poles of the machine. On taking up the glass one can detect no change with the eye or the microscope ; but when either side is breathed upon, a clear

* Read June 24, 1892.

frosted picture appears of that side of the coin which had faced it: even a sculptor's mark beneath the head may be read. For convenience those parts where the breath seems to adhere will be called white, the other parts black. In this experiment the more projecting parts of the coin have a black counterpart, but there is a fine gradation of shade to correspond with the depth of cutting in the device: the soft undulations of the head and neck are delicately reproduced.

The microscope shows that moisture is really deposited over the whole surface, the size of the minute water granulation increasing as the point of the picture is darker in shade.

There seems to be no change produced by the use of coins of different metals.

If sparking is allowed across the glass instead of at the poles of the machine, traces of metal are sometimes deposited beyond the disk of the coin, but not within it.

Around the disk is a black ring $\frac{1}{4}$ inch broad: sometimes the milling of the coin causes radial lines across this halo.

If carefully protected there appears to be no limit to the permanence of the figures, but commonly they are gradually obscured by the dust gathered up after being often breathed upon: some of the early ones, done more than two years back, are still clear and well defined in the detail.

It is possible to efface them with some difficulty by rubbing with a leather whilst the glass is moist. They are best preserved by laying several together when dry and wrapping them in paper: they are not blurred by this contact.

It is a curious fact that certain developments take place after a lapse of some weeks or months. The dark ring around the disk gradually changes into a series of three or four, black and white alternately; other instances of such a change will be noted below.

Let it be noticed that in coin pictures the object is near to, but not in contact with, the glass: for in the best specimens the rim of the coin keeps the inner part clear of the surface.

Obviously a small condenser is made by the coins: it is not essential; at the same time images made by a single coin, put to a single pole, are inferior.

The plan which gives the surest and most beautiful results is to place five or six coins, lying in contact side by side in a

cross or star, on either side of the glass: it is not necessary that each coin should exactly face one on the other side.

There has not appeared any distinction between the figures made by positive and negative electricity.

When several coins are side by side, touching one another, there appear in the spaces between them, which are mostly black, well-defined white lines, common tangents to the circular edges of the coins. If these are of equal size the lines are straight; otherwise they are curved, concave towards a smaller coin. They seem to be traces in that plane of the loci of intersection of equipotential surfaces.

Similar effects are obtained when coins and glasses are piled up alternately, and the outer coins are put to the poles of the machine. With six glasses and seven coins perfect images have been formed on both sides of each glass. With eight glasses the figures were imperfect; but there is little doubt this could be improved by continued trials as to the amount of electricity applied.

If several glasses are superposed and coins are applied to the outer surfaces, there are only the two images at the outside. After the electrification there is a strong cohesion between the plates.

It requires some practice to manage the electrification so as to produce the best results. There are two forms of failure which present interesting features. Sometimes a picture comes out with the outlines dotted instead of being continuous. At other times, if the electrification is carried too far, the impression comes out wholly black; but on rubbing the glass when dry with a leather the excess is somehow removed. Naturally it is difficult to rub down exactly to the right point, but I have succeeded on several occasions in developing from a blank all the fine detail of elaborate coins.

Here, again, we have another instance of the development by lapse of time, for an over-excited piece of glass usually gives a clear picture after an interval of a day or two.

Impressions from stereotype plates have been taken of which the greater part is legible: the distinctness usually improves after a few days. In default of a second plate, a piece of tin-foil about the same size should be put on the opposite side of the glass.

Sheet and plate glass of various thicknesses have been used without any noticeable change either in the treatment or the results.

I have put an impressed glass on a photographic plate in the dark, but did not get any result on developing: my imperfect skill in photographic matters leaves this experiment inconclusive.

Probably all polished surfaces may be similarly affected: a plate of quartz gives the most perfect images, which retain their freshness longer than those on glass.

Mica and gelatine give poorer results: it is not possible to polish the surface to the necessary point without scratching it.

On metal surfaces fairly good impressions can be produced if, as Karsten advises, oiled paper is put between the coin and the surface.

In the order of original discovery the figures noticed by Peter Riess should come first. He discusses a breath-track made on glass by a feeble electrical discharge; as well as two permanent marks, noticed by Ettrick, which betray a disintegration of the surface.

I have found that when a stronger discharge is employed more complex phenomena of a similar kind are produced. A 6-inch Wimshurst machine is arranged with extra condensers, as if to pierce a piece of glass. If this is about 4 inches square the spark will generally go round it. For a day, more or less, there is only a bleared watery track, $\frac{3}{10}$ inch wide, when the glass is breathed upon; but after this time others develop themselves within the first, a fine central black line with two white and two black on either side, the total breadth being the original $\frac{3}{10}$ inch. These breath-lines do not precisely coincide in position with the permanent scars, but the central one is almost the same as a permanent mark, which the microscope shows to be the surface of glass fractured into small squares of considerable regularity: on either side is a grey-blue line always visible, which Riess ascribes to the separation of the potash. After several months I found two blue lines on either side, which I believe were not visible at first. Of course these blue lines may be seen on most Leyden jars, where they have discharged themselves across the glass.

In 1842 Möser, of Königsberg, produced figures on polished surfaces by placing bodies with unequal surfaces near to them: the action was ascribed to the power of light, and his results were compared with those of Daguerre. Möser says, "We cannot therefore doubt that light acts uniformly on all bodies, and that, moreover, all bodies will depict themselves on others, and it only depends on extraneous circumstances whether or not the images become visible." In general, the multitude of images would make confusion; it can only be freshly polished surfaces that are free to reveal single definite impressions. However great Möser's assumption may be, there are many achievements of modern photography that would be as surprising if they were not so familiar. I have not the means of knowing the precise form of Möser's methods: in the experiments which follow there is usually contact and light pressure, and if they are not wholly analogous, they may for that cause help to generalize the idea: in none of these is electricity applied.

A piece of mica is freshly split, and a coin lightly pressed for 30 seconds on the new surface: a breath-image of the coin is left behind. At the same time it may be noticed that the breath causes abundant iridescence over the surface, whilst it is in a fresh state. It is not clear how the electricity of cleavage can have an active agency in the result.

It is familiar to most people that a coin resting for a while on glass will give an outline of the disk, and sometimes faint traces of the inner detail when breathed upon.

An examination-paper, printed on one side, is put between two plates of glass and left for ten hours, either in the dark or the daylight: a small weight will keep the paper in continuous contact, but this is not necessary if thick glass is used. A perfect breath-impression of the print is made, not only on the glass which lay against the print but also on that which faced the blank side of the paper. Of course the latter reads directly, and the former inversely; the print was about one year old, and presumably dry.

More often both impressions are white, sometimes one or other or both are black. At other times the same one may be part white and part black, and they even change while being examined.

During a sharp frost with east winds early in March, 1890, these impressions of all kinds were easy to produce, so as to be quite perfect to the last comma; but in general they are difficult, more especially those from the blank side.

At the best period those from the blank side of the paper were white and very strong; also there were white spots and blotches revealed by the breath. They seemed to correspond with slight variations in the structure of the paper, and suggest an idea that the thickness of the ink or paper makes a minute mechanical indentation on the molecules: the state of these is probably tender and sensitive under certain atmospheric conditions, as happens with steel in times of frost.

The following experiments easily succeed at any time:— Stars and crosses of paper are placed for a few hours beneath a plate of glass: clear white breath-figures of the device will appear. A piece of paper is folded several times each way to form small squares, then spread out and placed under glass: the raised lines of the folds produce white breath-traces, and a letter weight that was above leaves a latent mark of its circular rim.

Some writing is made on paper with ordinary ink and well dried: it will leave a very lasting white breath-image after a few hours' contact. If, with an ivory point, the writing is traced with slight pressure on glass, a black breath-image is made at once. Of course this reads directly, and the white one inversely. It is convenient to look through the glass from the other side for inverse impressions, so as to make them read direct.

Plates of glass lie for a few hours on a table-cover worked with sunflowers in silk: they acquire strong white figures from the silk.

In most cases I have warmed the glass, primarily for the sake of cleansing it from moisture; but I have often gone to a heat beyond what this needs, and think that the sensitiveness has been increased thereby.

It is not not easy to imagine what leads to the distinction between black and white, different substances act variously in this respect. I have placed various threads for a few hours under a piece of glass, which lay on them with light pressure: wool gives black, silk white, cotton black, copper white. A

twist of tinsel and wool gives a line dotted white and black; after a time these traces show signs of developing into multiple lines as in the spark figures.

Two cases have been reported to me where blinds with embossed letters have left a latent image on the window near which they lay; it was revealed in misty weather, and had not been removed by washing. I have not had a chance to see these for myself, but both my informants were accustomed to scientific observation.

A glass which has lain above a picture for some years, but is kept from contact by the mount, will often show on its inner side an outline of the picture, always visible without breath. It seems to be a dust figure easily removed: possibly, heat and light have loosened fine paint particles, and these have been drawn up to the glass by the electricity made in rubbing the outer side to clean it. The picture must have been well framed and sealed from external influences; most commonly dust and damp get in and obscure such a delicate effect.

I am not able to suggest simple causes for these varied effects. I am not inclined to think, except in the case of water-colours, which is hardly part of the inquiry, that there is a definite material deposit or chemical change; one cannot suppose that imperceptible traces of grease, ineradicable as they may be, would produce complete and delicate outlines. The cleaning off of impressions may at first seem to indicate a deposit; but this renewal of the surface might rather be like smoothing out an indented tin-foil surface: such a view might explain the case where a blank over-electrified disk is developed into fine detail. The electrified figures seem to point to a bombardment, which produces a molecular change, the intensity of electricity bringing about quickly what may also be done by slow persistent action of mechanical pressure. At present it seems as if most of the phenomena cannot be drawn out from the unknown region of molecular agency.

While experimenting I was not within reach of references to former researches, but I have since done my best to find them out, and to indicate all I have learnt in the body of my paper.

Poggendorff, vol. lvii. p. 492; translated in *Archives de l'Electricité*, 1842, p. 647.
Riess' *Electrische Hauchfiguren* in *Repertorium der Physik*; translated in *Archives de l'Electricité*, 1842, p. 591.
Riess' *Die Lehre von der Reibungs Electricität*, vol. ii. pp. 221–224.
Mascart, *Electricité Statique*, vol. ii. p. 177.
Taylor's Scientific Memoirs, vol. iii.

XLI. *Inductoscript.* *By* FREDERICK J. SMITH, *M.A., Millard Lecturer Mech. et Phys., Trinity College, Oxford* *.

THE well-known breath figures of Möser, Riess, and Karsten suggested to me the idea of producing similar figures on photographic plates; my laboratory day-book for 1878 shows that I tried an experiment to determine the action of a discharge from a coin on to a photo-plate. The image was produced on development, but very imperfectly. During the last two years an inductorium or transformer has been under constant trial in my laboratory. It occurred to me that probably its high potential might be used to produce figures similar to those of Karsten: plates of glass were cleaned with care, and operated on in the way suggested by Karsten, but no results were to be got out of it; probably the potential difference was not great enough.

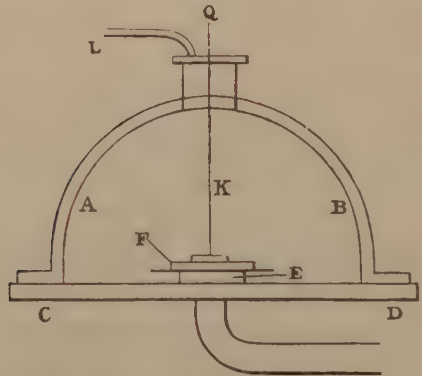
I then went on to place a photo-plate in the place of the glass, and on development the exact pattern of the medal used was produced; this showed that a PD, greatly under that of a jar discharge, or that of an electrical machine, would upset the chemical equilibrium of the photo-plate. Several factors appeared to be contributing to the result—the potential difference; the gas in which the experiment was performed; the pressure under which the experiment was done; the temperature; and the history of the plate previous to the experiment.

* Read June 24, 1892.

The following inductoscripts (a name I venture to propose, as it somewhat suggests the nature of the process, without being hybrid in construction), Nos. *A* to *W*, were produced under varying circumstances.

The experiments were performed under a bell-jar, A, B, fig. 1. This was placed on the air-pump plate, C, D; a round copper disc, E, was supported on the pump-plate on

Fig. 1.



three points; on this disc a sensitive plate, F, was placed, and on this the coin the print of which was to be produced. The electricity was introduced through the copper rod K, and the plate and rod were connected to the terminals of the inductorium, driven by four accumulators in series, each having seven plates $12'' \times 12''$.

By means of the curved tube, L, different gases were introduced into the bell-jar.

The intensity of the spark could be regulated by bringing the terminals of the inductorium together; when they were brought to within 4 mm. of each other the discharge was called *slight*, when placed wide apart it was called *full on* or *full*.

The following results were obtained:—

No.	Time of exposure (seconds).	Gas.	Electricity.	Pressure in receiver.
A.	10	Oxygen +	Pos. Slight	760
B.	10	Air +	Slight	760
C.	10	Air +	Full	760
D.	5	Oxygen +	Moderate	760

No.	Time of exposure (seconds).	Gas.	Electricity.	Pressure in receiver.
E.	10	Oxygen +	Moderate	760
F.	10	Oxygen +	Moderate	760
G.	30	Oxygen +	Slight	760
H.	5	Coal-gas +	Full	760
I.	20	Coal-gas −	Slight	760
J.	50	Air −	Neg.	760
K.	5	Air −	Full	760
L.	20	Air −	Slight	760
M.	5	Air −	Slight	760
N.	2	Air −	Slight	760
O.	5	Oxygen −	Moderate	760
P.	5	Air −	Moderate	760

Effects of a vacuum :—

No.	Time (seconds).	Gas.	Pressure (indicated by barometer-tube on pump).
Q.	5	Air	745 mm.
R.	5	Air	745 mm.
S.	5	Air	755 mm., point of conductor 2·5 cm. above coin, ½ sovereign.
T.	5	Air	630 mm., point ½ cm. above coin.
U.	5	Air	755·5 mm., point ½ cm. above coin.
V.	5	Oxygen	700, point ½ cm. above coin.
W.	10	Air	760, plate exposed to daylight before being used, the image developed out, but the whole plate quickly turned black all over.
X.	10	Oxygen	760.

It will be noticed that in *V* and *X* the discharge has produced the peculiar effect of transparent streaks in one direction, and black opaque streaks in an opposite direction. During the development the black streaks came out first, and some time afterwards the transparent ones. Experiments bearing on this point will be repeated.

It is evident from the inductographs taken *in vacuo* that a good vacuum materially checks, and nearly stops, the phenomenon taking place.

It is to be noticed in reading the work on breath figures by Grove and Karsten, that Grove tried to set the image by coating an electrised plate with collodion and nitrate of silver, and then developed out the word Volta (sec. 4, p. 404, 'Correlation of Physical Forces,' Grove). He did not let

the electrical discharge act on the silver salt. Again, Karsten did not succeed in producing a fixed picture on a daguerreotype plate ; he writes, "If an iodized plate is taken, and a picture produced thereon, the vapour of mercury will not bring it out, although the image is really on the plate, as may be shown by breathing upon it." Möser appears to have got out pictures by means of water-vapour, but they were not permanent effects.

As a summary of my own work, I would mention that—

I. The different gases used were oxygen, carbonic acid gas, air, coal-gas. Far the best results were obtained in oxygen. Experiments with other gases are now being made.

II. The effect of pressure. No picture could be obtained in a good vacuum, only the circular shape of the coin ; as as the pressure increased towards the normal the pictures became more perfect.

III. The output of the transformer was regulated by resistances put into the primary circuit, also by the rate at which the reversing commutator was driven ; the reversing commutator is similar to those used on the early Siemens machine, and driven from a belt in the laboratory.

IV. The temperature varied between 5° C. and 100° C. Perhaps the effects may have been produced more quickly at the higher temperature, but no significant change was apparent.

V. Similar experiments are about to be performed in many other gases, and under considerable pressure, up to 10 atmos. The apparatus is nearly ready.

VI. In addition to pictures on photo-plates, good impressions have been obtained on bromide paper and other papers direct. Pictures can easily be got from woodcuts, after they have been well covered with plumbago.

XLII. *On the Relation of the Dimensions of Physical Quantities to Directions in Space*.* By W. WILLIAMS, *Assistant in the Physical Laboratory, Royal College of Science†.*

IN a paper read before the Physical Society, Nov. 24, 1888, Prof. Rücker showed that in the dimensional formulæ of electromagnetic quanties μ and k the two specific capacities of the medium are generally omitted, or rather their dimensions are suppressed, and that the consequence of this suppression is to give rise to two artificial systems of dimensions, the electrostatic and the electromagnetic. That these systems are artificial appears when we consider that each, apparently, expresses the *absolute* dimensions of the different quantities, that is their dimensions only in terms of L, M, and T; whereas we should expect that the absolute dimensions of a physical quantity could be expressed only in one way. Thus, from the mechanical force between two poles, we get

$$f=\frac{1}{\mu}\left(\frac{m^2}{r^2}\right), \qquad \therefore \quad m=r\sqrt{\mu f}\,;$$

and this, being a qualitative as well as a quantitative relation, involves the dimensional identity of the two sides. If now μ is put $=1$, we either *ignore* its dimensions or *assume* that it has none, being but a mere number. In the former case the dimensions deduced for m under such circumstances must be admittedly artificial, since if the suppressed dimensions of μ were inserted, those of m would be different, and would involve a different physical interpretation. In the latter case the dimensions deduced for m must be its absolute dimensions and must therefore involve its physical interpretation. But if we start with the relation

$$f=\frac{1}{k}\left(\frac{q^2}{r^2}\right), \qquad \therefore \quad q=r\sqrt{kf},$$

where f is now the force between two charges q, and if we

* See Note at the end of the Paper.

† Read June 24, 1892, and communicated by A. W. Rücker, F.R.S.

treat k as a pure number, we obtain for m by means of the dimensions of q deduced by this relation absolute dimensions again, but different now to what we had before. In this way we get two different absolute dimensions for the same physical quantity,—each of which involves a different physical interpretation. If, however, μ and k be not mere numbers, but concretes expressing physical properties of the medium, as is indeed proved by the relation

$$[\mu k] = (L^{-2}T^2)^*,$$

then the fact that the dimensions of electrical and magnetic quantities are different in the two systems arises only from our ignorance of the dimensions of μ and k, which must be such as to bring the two systems into accord when ultimately expressed in terms of L, M, and T. Prof. Rücker therefore suggested that μ and k should be admitted into the formulæ along with L, M, and T, and that, since nothing is definitely known as to their physical nature, they should be regarded as secondary fundamental units, additional to L, M, and T, but which must be ultimately expressed in terms of them. There is thus, in each formula, an indication that the irrational and unsuggestive dimensions are not absolute as they stand; each formula is made to express a physical reality, and each contains in the proper form the factor necessary to render the formula *absolute*. This system has been adopted, among others, by Prof. Gray in his smaller edition of 'Absolute Measurements in Electricity and Magnetism,' and by Prof. Everett in his last edition of 'Units and Physical Constants.'

Prof. Fitzgerald has pointed out (Phil. Mag. April 1889) that if a system be taken "in which the dimensions of μ and k are the same, and of the dimensions of a slowness, that is the inverse of a velocity $(L^{-1}T)$, then the two systems become identical as regards dimensions, and differ only by a numerical coefficient just as centimetres and kilometres do." But although μ and k are quantities of the same order, both being capacities, their dimensions need not necessarily be the same, unless electrification and magnetization be phenomena of the

* See also "On the Dimensions of a Magnetic Pole in the Electrostatic System of Units," Dr. Lodge, Phil. Mag. Nov. 1882.

same order as well. If, however, these be different orders—as they almost certainly are—the one possibly a strain, the other a vortex motion, then it is unlikely that k and μ, bearing, as each does, similar relations to two dissimilar phenomena, should have identical dimensions.

In one of the Appendices to 'Modern Views of Electricity' Dr. Lodge develops a system in which μ and k^{-1} have respectively the dimensions of *density* and *rigidity*. It is then found that the dimensions of all the other quantities become unique, and capable of purely dynamical interpretations. Thus, magnetic moment becomes linear momentum; magnetic induction, linear momentum per unit volume; magnetic force, velocity; electrical force, pressure; current, displacement × velocity; self-induction, "inertia per unit area," &c. In a similar manner other such systems may be developed. Thus, take the equation

$$F=M\frac{\partial v}{\partial t}+kv,$$

where F is an impressed force, M the mass of a moving body, v its velocity, and k a coefficient of resistance. Compare this with

$$E=L\frac{\partial C}{\partial t}+RC,$$

where E is the voltage of a closed circuit, L its self-induction, C the current, and R the resistance. Let us identify E and F dimensionally, and work out the analogy in detail. To do this, we have only to equate the dimensions of E and F, and find what value of μ satisfies the relation. Then substituting this value of μ in the formulæ of the other quantities, we are able to complete a connected dynamical analogy of electromagnetism by starting with *voltage* as a *force*. In this case, we find that electrification is a displacement; current, a velocity; electrical potential, force; quantity of magnetism, linear momentum; self-induction, inertia, &c. In a precisely similar manner, by starting with the equation

$$G=I\frac{\partial \omega}{\partial t}+B\omega,$$

where G is a couple, I a Moment of Inertia, and ω an angular

velocity, we get:—electrification, a strain; electrical potential, work; electrical force, force; current, angular velocity; quantity of magnetism, angular momentum; self-induction, moment of inertia, &c. In all cases we are able, by means of the dimensional formulæ expressed in terms of μ and k, to work in detail any dynamical analogue we may choose to take as a starting-point. Of course all dimensional values of μ, together with the corresponding ones for k, must render electromagnetic dimensions unique. It is only some of these, however, that give rise to formulæ capable of dynamical interpretation, and it will be found on examination that the number of such values is small. Now, if electromagnetism is ultimately dynamical, the dimensions of electromagnetic quantities must be of the same kind (ultimately) as those of ordinary dynamical units. Hence, by examining all the possible cases in which the dimensions of μ and k lead in the case of the other electro-magnetic quantities to dimensions of the dynamical order, we *may* be able to obtain some light as to the nature of μ and k themselves. To show how this may be done, and the kind of results obtained will be the object of the following paper.

Before doing this, however, it becomes necessary to examine in more detail the real nature of dimensional equations, and to determine how far they are capable of giving reliable results when used in the way here suggested. Primarily, a dimensional formula expresses only numerical relations between units, and for the purpose of the present paper is defective from the fact that different physical quantities have the same dimensional formulæ. For example, *couple* and *work*, as pointed out by Prof. S. P. Thompson in the course of the discussion on Prof. Rücker's paper above referred to; or, again, an *area* and the *square of the same vector-length*; *pressure* and *tangential force per unit area*, &c. If, therefore, dimensional equations are to be used at all in the sense above indicated, it becomes necessary, in the first place, to be assured that the process is valid, and, in the second place, that no contradictory or unintelligible results arise from causes such as the above.

The dimensional formula of a physical quantity expresses

the *numerical* dependence of the unit of that quantity upon the fundamental and secondary units from which it is derived, and the indices of the various units in the formula are termed the *dimensions* of the quantity with respect to those units. When used in this very restricted sense, the formulæ only indicate numerical relations between the various units. It is possible, however, to regard the matter from a wider point of view, as has been emphasized by Prof. Rücker in the paper above referred to. The dimensional formulæ may be taken as representing the *physical identities* of the various quantities, as indicating, in fact, how our conceptions of their physical nature (in terms, of course, of other and more fundamental conceptions) are formed—just as the formula of a chemical compound indicates its composition and chemical identity. This is evidently a more comprehensive and fundamental view of the matter, and from this point of view the primitive numerical signification of a dimensional formula as merely a change ratio between units becomes quite a dependent and secondary consideration.

The question then arises, what is the test of the identity of a physical quantity? Plainly, it is the manner in which the unit of that quantity is built up (ultimately) from the fundamental units L, M, and T, and not merely the manner in which its magnitude *changes* with those units. Thus, the unit couple and the unit of work both change in the same manner with the unit length, but they are physical quantities of different kinds. Their *numerical* dependence upon L, M, and T is the same, as expressed by the formula $(MLT^{-2})L$, but the manner the unit length enters their definition is different: in the case of work the two units of length involved are in the same direction; in the case of couple they are mutually at right angles. The absolute measure of a force is the work done through unit linear displacement; similarly, the absolute measure of a couple is the work done through unit angular displacement. Hence the relation between couple and work is similar to that between force and work, the difference being that angular displacements are considered in the former case, linear displacements in the latter. But the measure of an angular displacement is independent of the unit length. Hence, in expressing the numerical dependence of

the unit couple and the unit of work upon the fundamental units we get the same formula ML^2T^{-2}, although the difference in the physical nature of the quantities is of the same kind as that between force and work. And generally, when used in the purely numerical sense above indicated, the dimensional formulæ fulfil all requirements; it is only when endowed with the higher function of defining the physical identities of the various quantities that they are found to fail.

That the dimensional formulæ *are* regarded from this higher standpoint—that is, regarded as being something more than mere "change ratios" between units—is shown by the fact that difficulties are felt when the dimensions of two different quantities, *e. g.*, couple and work, happen to become the same. If, however, the numerical dependence of the units of the two quantities upon the fundamental units be the same, and if the formulæ are to express nothing more, then the two quantities *must* have the same dimensions, and from this point of view we are not entitled to feel any difficulty in the matter. That such difficulties are felt arises therefore from the more comprehensive signification which is attached to the formulæ—a signification which obviously includes all the numerical considerations which alone constitute the more restricted one.

Let us, therefore, for the purpose of the present paper, regard the dimensional formula of a quantity as the symbolical expression of the physical nature of that quantity, so far, of course, as it depends upon the fundamental conceptions of mass, space, and time (and, in the case of thermal and electrical quantities, of secondary conceptions also ultimately dependent upon mass, space, and time). To obtain the formula for any quantity, it is only necessary to express how the unit of the quantity is built up from the fundamental and secondary units. Now, the units of mass and time, and all secondary units, are involved in all physical units in a simple manner. They are raised to different powers. But, owing to the dimensions of space, the unit of length is involved in different ways, according to the different relative directions in which it may be taken. In all cases, however, the unit of any quantity can be completely expressed—so far as it involves the unit length—by taking the unit length along one

or more of three mutually rectangular directions Ox, Oy, Oz, whose absolute direction in space is of course determined by the nature of the physical relation into which the quantity enters. The dimensional formulæ can therefore be expressed in terms of M, T, and X, Y, Z, where M is the unit of mass, T the unit of time, and X, Y, Z the unit of length taken respectively along the directions Ox, Oy, Oz. Thus, if MXT^{-2} is the unit of force, MX^2T^{-2} is the unit of work; $MXY^{-1}Z^{-1}T^{-2}$, energy per unit volume; $MXZT^{-2}$, a couple in the plane XZ, &c. The above quantities are, of course, the same in kind but different in direction from MYT^{-2}, MZT^{-2}, &c. This method of expressing the dimensional formulæ is nothing more than a modified extension of Prof. S. P. Thompson's suggestion as to the use of $\sqrt{-1}$ in dimensional formulæ, as will appear more fully later (see discussion on Prof. Rücker's paper above referred to).

When it is necessary to determine the *numerical* dependence of the unit of a quantity upon the fundamental and secondary units, the distinction between units of length taken in different directions must be suppressed, and the unit of length in whatever direction taken must be represented by a single symbol L. The index of L will therefore be the *sum* of the indices of X, Y, and Z, and the formulæ thus simplified will indicate the changes in the magnitude of the unit when the fundamental and secondary units are themselves changed. Thus, we can immediately deduce from any formula the particular and simplified form it assumes when only the numerical relations between units are to be considered. It will be convenient to designate these simplified forms of the formulæ the "change ratios" of the units, and to reserve the term "dimensional formulæ" for the more general forms in which the identities of the quantities are primarily concerned.

The dimensions of the ordinary dynamical quantities may now be expressed:—

1. Inertia = M.
2. Length = X, Y, or Z.
3. Area = XY, YZ, or ZX.
4. Volume = XYZ.
5. Density = $MX^{-1}Y^{-1}Z^{-1}$.

6. Velocity $= XT^{-1}, YT^{-1}, ZT^{-1}$.
7. Acceleration $= XT^{-2}, YT^{-2}, ZT^{-2}$.
8. Linear momentum $= MXT^{-1}, MYT^{-1}, MZT^{-1}$.
9. Force $= MXT^{-2}, MYT^{-2}, MZT^{-2}$.
10. Work $= MX^2T^{-2}, MY^2T^{-2}, MZ^2T^{-2}$.
11. Energy per unit volume $= MXY^{-1}Z^{-1}T^{-2}$, $MX^{-1}YZ^{-1}T^{-2}$, $MX^{-1}Y^{-1}ZT^{-2}$.
12. Pressure $= MXY^{-1}Z^{-1}T^{-2}$, [Force $= MXT^{-2}$, per unit of surface YZ] &c.

Thus, the dimensions of pressure are the same as those of energy per unit volume, as should be the case, for the pressure at a point in a gas is given by $p=\frac{1}{3}\rho V^2$, where ρ is the density and ρV^2 is of the dimensions of energy per unit volume. The case is different from that of $W=G\theta$, where W is the work done by a couple G through an angular displacement θ. In the former case, p is the same in *kind* as ρV^2, nothing concrete intervening. In the latter case, G cannot be the same as W unless θ be a mere number, having no reference to anything concrete, which is not the case.

13. Couple $= MXYT^{-2}$, (Force along X *or* Y, Arm Y *or* X) &c.

The rational measure of an angle is $\frac{s}{r}$, where s is the arc described by the radius r rotating about one extremity. Let ∂s be an element of the arc, then $s=\Sigma\partial s$, and $\theta=\frac{\Sigma\partial s}{r}$, or $\partial\theta=\frac{\partial s}{r}$. Let r be measured along X, then, since ∂s is always an element at right angles to r, ∂s will be measured along Y or Z. If these directions (X, Y, Z) be carried along *with* r—that is, if we take instantaneous axes at every point of the arc—the axes bearing the same relation to the radius and tangent at every point, we get $\partial\theta=\frac{\partial y}{x}$, and $\theta=\frac{\Sigma\partial y}{x}$. To express this dimensionally, we have to neglect the summation Σ; for a dimensional formula expresses the *nature* of a quantity, not its magnitude, and the same formula must therefore apply to both θ and $\partial\theta$. The dimensions of θ and $\partial\theta$ are therefore YX^{-1}.

14. Angles $= XY^{-1}$, (Direction of radius Y, of the arc X) &c.
15. Angular Velocity $= XY^{-1}T^{-1}$, &c.
16. Angular Acceleration $= XY^{-1}T^{-2}$.
17. Moment of Inertia $= MX^2, MY^2, MZ^2$.

$$\frac{\text{Moment of couple}}{\text{Angular acceler.}} = \frac{MXYT^{-2}}{XY^{-1}T^{-2}} = MY^2, \text{ \&c.}$$

18. Angular Momentum $= I\omega = MY^2(XY^{-1})T^{-1}$ $= MXYT^{-1}$, &c.
19. Energy of Rotation $= \frac{1}{2} I\omega^2 = MY^2(XY^{-1})^2T^{-2}$ $= MX^2T^{-2}$, &c.
20. Couple $= I\frac{\partial\omega}{\partial t} = MY^2(XY^{-1})T^{-2} = MXYT^{-2}$.
21. Work done by a couple $= G\theta = (MXYT^{-2})(XY^{-1})$ $= MX^2T^{-2}$.
22. Solid angle $= \partial\Omega = \frac{\partial a}{r^2}$, where ∂a is an element of area on the surface of a sphere radius r. Taking instantaneous axes, X being always along the radius, and Y, Z in the tangent plane at a point, this becomes dimensionally,

$$[\partial\Omega] = (YZX^{-2}).$$

The quantity π may enter into physical relations in two different ways. It may enter in its purely geometrical or trigonometrical capacity as a definite number of radians (or, what is equivalent, as the ratio of the circumference of a circle to its diameter), and be thus definitely related in a physical sense to the other quantities ; or it may enter in its numerical capacity as part of a numerical coefficient determined by higher abstract analysis. Cases of the former kind only have to be considered in the following paper. In these cases π, enters the relations (ultimately) as a definite number of plane or solid angles, and it therefore consists of two factors, namely, a numerical factor, 3·14···, and a concrete factor, the unit plane or solid angle. We may therefore speak of the dimensions of π, meaning thereby the dimensions of the plane or solid angles which it *in such cases* implies, and the dimensional identity (in the directional sense) of the relations into which it so enters cannot be complete if the concrete factor is

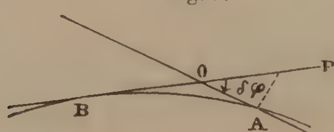
neglected. Thus, let $A=4\pi r^2$, where A is the surface of a sphere of radius r. Taking axes as in 22, this becomes dimensionally $YZ=(YZX^{-2})X^2$, and the relation is not true if 4π be treated as a pure number unless an area is represented by the square of a vector length, instead of by the product of *two* different vector lengths as in vector algebra. Similarly, in the relation $V=\frac{4}{3}\pi r^3$, V being a volume, π is of the dimensions of a solid angle, while in $l=2\pi r$, where l is the circumference of a circle, it is of the dimensions of a plane angle.

The significance of π as it occurs in electromagnetism in connexion with radial and circuital fluxes will be considered later.

The following examples serve to illustrate this method of expressing dimensional formulæ.

23. *Radius of Curvature.*—Bending per unit length of a

Fig. 1.



curve. Let A B be an element of a curve, and let the tangents at A and B intersect at O, making an angle $\partial\phi$. The angle $\partial\phi$ is ultimately $=\dfrac{AP}{OP}$. If A P be taken along the normal X at A, and O A along the tangent Y, then A B is also ultimately along Y, and may be written ∂y. Hence the curvature becomes $\dfrac{\partial\phi}{\partial y}$, or dimensionally $(XY^{-1})Y^{-1}=XY^{-2}$, where XY^{-1} are the dimensions of $\partial\phi$, and Y those of AB. The radius of curvature at O is therefore of the dimensions (Y^2X^{-1}).

24. *Centrifugal Force.*—Let a particle describe the path A B with velocity V, the centrifugal force is $F=\left(m\dfrac{V^2}{\rho}\right)$. Taking instantaneous axes at A, as in 23, this becomes dimensionally

$$[F]=M(Y^2T^{-2})(XY^{-2})=MXT^{-2}.$$

Thus the force is directed along the radius X.

25. *Compressibility.*—Hydrostatic pressure is of the dimensions $MXT^{-2}(YZ)^{-1}$, according to the direction of the plane of reference YZ. Strain is here of no dimensions, being the ratio of two concretes of the same kind. Hence the dimensions of compressibility are $M^{-1}X^{-1}YZT^2$.

26. *Rigidity.*—"A simple shear is a homogeneous strain in which all planes parallel to a fixed plane are displaced in the same direction parallel to that plane and therefore through spaces proportional to their distances from that plane" (Kelland and Tait's 'Quaternions,' p. 204). Let the planes of displacement be the planes XY, and the direction of displacement X. The applied stress (tangential force per unit area) is

$$\frac{MXT^{-2}}{XY}=MY^{-1}T^{-2}.$$

The shear is XZ^{-1}. Hence the rigidity is of the dimensions

$$\frac{MY^{-1}T^{-2}}{XZ^{-1}}=(MX^{-1}Y^{-1}ZT^{-2}).$$

The velocity of propagation of a wave of distortion is

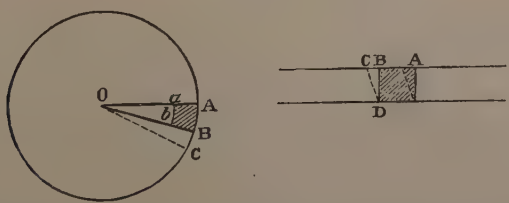
$V=\sqrt{\frac{n}{\rho}}$, $n=$ rigidity. Hence

$$[V]=\sqrt{\frac{MZ(YX)^{-1}T^{-2}}{M(XYZ)^{-1}}}=ZT^{-1}.$$

Thus, the applied stress acts over the face XY, and the disturbance is propagated along Z.

27. *Torsion.*—Let OABC be the section of a cylinder, axis Z and radius Y, subjected to torsion; $aABb$ the face of an elementary cube edge ∂r, bounded as in the figure; and θ the twist per

Fig. 2.



unit length. The shear experienced by the elementary cube is ultimately $\frac{BC}{BD}=r\theta$, or dimensionally XZ^{-1} (for θ is of the

dimensions $XY^{-1}Z^{-1}$, and $r=Y$). Let $P=nr\theta$ be the applied stress (tangential force per unit area). Then $[P]=MY^{-1}T^{-2}$. The tangential force over ring of which $aABb$ forms part $=(2\pi r\partial r)nr\theta=2\pi r^2n\theta\partial r$, or dimensionally $MY^{-1}T^{-2}(XY)=MXT^{-2}$, and the moment of this round $O=2\pi r^3n\theta\partial r$, or dimensionally $MXYT^{-2}$. The moment due to the whole face

$$=2\pi n\theta\Sigma_0^R r^3\partial r=\left(\frac{\pi nR^4}{2}\right)\theta,$$

where $R=OA$, and this is of the same dimensions as $2\pi nr^3\theta\partial r$, for the r^4 in the former expression is of the same dimensions as $r^3\partial r$ in the latter. Thus $\frac{\pi nr^4\theta}{2}$ is of the dimensions $MXYT^{-2}$.

To determine from this the dimensions of π we have

$$\begin{aligned}
[\pi nr^4\theta] &= MXYT^{-2},\\
[\theta] &= XY^{-1}Z^{-1},\\
[r^4] &= Y^4,\\
[n] &= MZ(XY)^{-1}T^{-2};
\end{aligned}$$

$$\therefore\ [\pi]MZ(XY)^{-1}T^{-2}\,.\,Y^4\,.\,(XY^{-1})Z^{-1}=MXYT^{-2},$$

$$\therefore\ [\pi]=XY^{-1}.$$

Thus π in this case is of the dimensions of a plane angle in the plane XY.

28. *Viscosity.*—The viscous resistance between planes moving with relative velocity $u=\frac{\partial x}{\partial t}$ is given by $F=\frac{1}{4}\left(Nm\omega L\frac{\partial u}{\partial z}\right)$ (Clausius). Taking the components of ω and L along Z (normal to direction of motion) this becomes dimensionally

$$[F]=\left(\frac{M}{XYZ}\right)\left(\frac{Z}{T}\right)(Z)\left(\frac{X}{ZT}\right)=MY^{-1}T^{-2}=\frac{MXT^{-2}}{XY},$$

a tangential force in the plane of motion (XY). The coefficient of viscosity is

$$\eta=\frac{F}{\frac{\partial u}{dz}}=\frac{MY^{-1}T^{-2}}{XZ^{-1}T^{-1}}=MZ(YZ)^{-1}T^{-1}$$

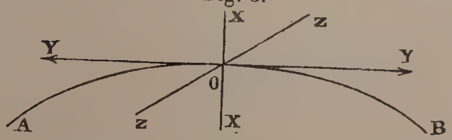
$=$Tangential force per unit area $\div$ shear per unit time.

29. *Surface Tension.*—Tangential force per unit length normal to itself. Let Y be the direction of force in the plane

YZ, and K the surface tension. Then

$$[K]=MYZ^{-1}T^{-2}=\frac{MY^2T^{-2}}{YZ}$$
$$=\text{Energy per unit area.}$$

Fig. 3.



Let AOB be a normal section of a cylindrical liquid film, axis Z, radius X. The normal pressure due to the curvature is $K\left(\frac{1}{\rho}\right)$, or dimensionally

$$MYZ^{-1}T^{-2}(XY^{-2})=MX(YZ)^{-1}T^{-2};$$

as should be the case.

The above examples are sufficient to illustrate the method of expressing dimensional formulæ in terms of X, Y, Z. In all the above cases isolated quantities only are dealt with, and therefore the relation of the directions of directed quantities to the dimensional axes is a matter of indifference. This is not so in the case of *equations* between quantities. Here, however, we have only to transform the equations to Cartesian co-ordinates and then express the dimensions of each term by means of the above. Thus, let A, B, C, . . . be quantities connected by the equation

$$A+B+C+D+\ldots=0.$$

Expressed in Cartesian co-ordinates this becomes

$$(A_x+A_y+A_z)+(B_x+B_y+B_z)+(C_x+C_y+C_z)+\ldots=0;$$

and we can now immediately express the dimensions of every term. We thus get the dimensional expression of the equation itself. A physical equation implies that the quantities connected are the *same in kind*. There must, therefore, be a dimensional identity between the various terms. This holds, however, in its wider or directive sense only of component quantities in the same direction, for if we equate any term in the above to all the rest we always get a term such as A_x

equated to another such as A_y, the same in kind, but different in direction. If, however, we write the equation

$$(A_x+B_x+C_x+\ldots)+(A_y+B_y+\ldots)+(A_z+B_z+\ldots)=0,$$

there must now be a dimensional identity in the directional (X, Y, Z) sense between the terms in the same bracket.

Up to the present the dimensional formulæ have been regarded as purely conventional. It is now necessary to examine whether the conventions made use of can be justified in any other way; and it becomes important to have a clear *physical* distinction between pure numbers and concretes, for, according to the above conventions, dimensional formulæ are now extended to include quantities hitherto regarded as pure numbers.

Every concrete quantity should have definite physical dimensions in the extended sense of the term; only pure numbers should be quantities of no dimensions. For physical purposes, a pure number may be defined as the ratio of two concretes of the same kind. If the concretes are directed quantities, their ratio is not a pure number unless they are *similarly directed.* Thus, the ratio of a force along X to that along Y is of the dimensions $(MXT^{-2})/(MYT^{-2})=XY^{-1}$; that is, of the dimensions of an angle, or rotation, indicating that to compare the scalar magnitude of the two forces, they have to be rotated into coincidence. And, generally, the ratio of any two directed quantities of the same kind is a pure number together with a quantity of the nature of rotation, the latter being the *versor* part of the quotient of two vectors. Since the measure of a rotation is independent of the unit length, the versor part of the quotient is a pure number so far as the scalar unit length is concerned. It cannot, however, be truly a pure number, for different degrees of rotation are compared in terms of definite units of rotation. Hence, a *versor*, or its equivalent, an angle, or an angular displacement is a concrete quantity equally well with mass, length, and time, and should have its own proper dimensions. It is only because of the restriction of the term "concrete" to quantities the magnitudes of whose units *change* with the units of length, mass, and time that concretes of the nature of angles, and angular displacements, come to be regarded as purely numeric. From

this point of view, therefore, a pure number is always of the nature of a *tensor*, or rather, unity must be regarded as the dimensional formula of a *tensor*, and of physical quantities of like nature, e. g. *volume-strains* *.

Let α, β, γ be three vectors (non-coplanar). Then

1. α, β, or γ = Linear displacements, or lengths in magnitude and direction.

2. $V(\alpha\beta)$ &c. = Area of parallelogram bounded by α, β.

3. $S(\alpha\beta\gamma)$ = Volume (neglecting sign) of parallelopiped defined by α, β, γ.

4. $U\frac{\alpha}{\beta} = Versor\frac{\alpha}{\beta}$ = Angular displacement required to bring β parallel to α.

If α, β, γ be vectors mutually at right angles, it is unnecessary to distinguish between scalar and vector products, and if they be also equal, between tensors and versors, for the product of any two vectors is now a vector, and of three a scalar: the quotient of any two is a versor, since the tensor $=1$; while the product of parallel vectors is always scalar. We may therefore take α, $\alpha\beta$, $\alpha\beta\gamma$, $\frac{\alpha}{\beta}$, and $\frac{\alpha}{\alpha}$ (or *unity*) as the typical representatives of the respective ideas of vector lengths, areas, volumes, angles, and pure numbers (*tensors*). If, now, we call these the dimensional formulæ (that is, the physical representations) of these ideas, we see that the tensor, although a *quasi*-physical quantity like the versor, becomes physically represented according to the above conventions by unity, while the versor is represented by the ratio of two vectors. Thus, the meaning of the fact that *tensors* and physical quantities of like nature are of no dimensions, is that their dimensional representation according to the above conventions simplifies down to unity. No other physical quantities are, however, dimensionally represented by unity. We may, therefore, say that *tensors* &c. are also physical quantities, and therefore concrete, and that since every physical quantity consists of two factors, namely a pure number and a concrete, we may regard a pure number when occurring alone in a physical relation as a physical quantity whose concrete factor is a *tensor*. Since, however, the term "concrete" is

* See "The Multiplication and Division of Concrete Quantities," Prof. A. Lodge, 'Nature,' July 19, 1888.

reserved for those quantities which cannot be completely specified by pure numbers, we come ultimately to regard a tensor as a pure number, and the versor as a concrete quantity.

If we substitute for α, β, γ in the above X, Y, Z respectively, we get the same expressions for lengths, areas, volumes, and angles as already obtained in the dimensional formulæ. Hence, we see that the conventions made as to the dimensional representation of the various quantities, namely, that areas, volumes, and angles should be represented by products and quotients of *different* vector lengths instead of by powers and quotients of a *given one*, are justified by the fact that they are consistent with the meanings of products and quotients of rectangular vectors. For the products and powers of parallel vectors can never represent areas or volumes, and their ratios can be nothing but pure numbers.

Dimensional formulæ may be conveniently expressed in this way, that is, by taking three rectangular vectors in space as axes of reference. We may, of course, suppose X, Y, Z to be the vectors. Then the formulæ already deduced will be unaltered, except that the proper signs have to be inserted according to the order the products and quotients are performed. The great advantage of thus supposing X, Y, Z to be vectors instead of mere Cartesian lengths is that the distinction between scalar and vector quantities is made apparent by the formula. Thus, X^2, Y^2, Z^2, or R^2 (R being any vector) are *scalars*; X, Y, Z, XY, YZ, ZX, XY^{-1}, YZ^{-1}, ZX^{-1} and their reciprocals are *vectors*; and (XYZ) a scalar. To make the formulæ exhibit at the same time the numerical dependence of the derived upon the fundamental units, we may put $X=i\mathrm{L}$, $Y=j\mathrm{L}$, $Z=k\mathrm{L}$, i, j, k being quadrantal versors and L the *scalar unit length*. Thus, neglecting signs, we have :—

$$\text{Force} = \mathrm{MXT^{-2}} = \mathrm{M}(i\mathrm{L})\mathrm{T^{-2}}.$$

$$\text{Work} = \mathrm{MX^2T^{-2}} = \mathrm{M}(i\mathrm{L})^2\mathrm{T^{-2}} = \mathrm{ML^2T^{-2}}.$$

$$\text{Couple} = \mathrm{MXYT^{-2}} = \mathrm{M}ij\mathrm{L^2T^{-2}} = \mathrm{M}k\mathrm{L^2T^{-2}}.$$

$$\text{Energy per unit volume,} = (\mathrm{ML^2T^{-2}})\mathrm{L^{-3}} = \mathrm{ML^{-1}T^{-2}},$$
$$\text{for volume} = (ijk\mathrm{L^3}) = \mathrm{L^3},\ ijk = 1.$$

$$\text{Pressure} = \frac{MXT^{-2}}{YZ} = \frac{MiLT^{-2}}{jkL^2} = ML^{-1}T^{-2}, \ (i = jk).$$

$$\text{Tangential force per unit area} = MY^{-1}T^{-2} = M(jL)^{-1}T^{-2}.$$

$$\text{Angle} = XY^{-1} = (iL)(jL)^{-1} = \frac{i}{j} = \frac{ij}{j^2} = k.$$

$$\text{Work done by a couple} = MXYT^{-2}(XY^{-1}) = (MijL^2T^{-2})k$$
$$= ML^2T^{-2}, \ (ijk = 1).$$

Thus, the indices of M, L, and T indicate the *numerical* dependence of the derived upon the fundamental units, the formulæ being identical in this respect with the ordinary ones, while the directional properties of the quantities are clearly differentiated by i, j, k.

Multiplying the numerator and denominator of a formula by X, Y, or Z makes no change in its physical interpretation. For physical quantities differ from each other only in their numerical dependence upon the fundamental units, and their "space relations," the former being expressed by the scalar part of the formula L, M, T, the latter by the vector part i, j, k. But if X, Y, Z be equal rectangular vectors, the tensors of $\frac{X}{X}, \frac{Y}{Y}, \frac{Z}{Z}$ are unity, and their versors are zero. Hence, no change can be made in the scalar or vector part of any formula into which they may be multiplied. Thus:—

$$\text{Force} = MXT^{-2} = \frac{MX^2T^{-2}}{X}$$

$$= \text{Space rate of variation of energy along X.}$$

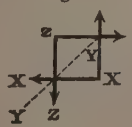
$$\text{Force} = MXT^{-2} = \frac{MXZT^{-2}}{Z} = \text{Couple per unit area.}$$

$$\text{Tangential force per unit area} = MY^{-1}T^{-2} = \frac{MXT^{-2}}{XY}$$

$$= \text{Force along X in plane } XY = \frac{MZT^{-2}}{YZ}$$

$$= \text{Force along Z in plane YZ.}$$

Fig. 4.



These are the two components of a shearing-stress referred to a unit cube. Multiplying by $\frac{XZ}{XZ}$ we get $\frac{MXZT^{-2}}{XYZ}$ = Couple per unit volume.

Now, a shearing-stress must be of the nature of a couple, for a shear is of the nature of an angle, and the product of the stress into the shear is work done per unit volume.

$$\text{Surface tension} = \frac{MXT^{-2}}{Y} = \frac{MX^2T^{-2}}{XY} = \text{Energy per unit area.}$$

$$\text{Pressure} = \frac{MXT^{-2}}{YZ} = \frac{MX^2T^{-2}}{XYZ} = \text{Energy per unit volume.}$$

Since, as above shown, we obtain the same dimensional formulæ by Cartesian and vectorial methods, we may attach to X, Y, Z and their products and quotients in these formulæ either purely vectorial or purely Cartesian meanings just as we please: in both cases, the formulæ represent the same physical identities. When X, Y, Z are vectors the formulæ express the directional properties of the corresponding quantities. [An inspection of a formula shows this, for the products and quotients of two rectangular vectors are vectors directed normally to the planes containing the original vectors, and the products and quotients of parallel vectors are scalar]. Thus: Pressure $MX(YZ)^{-1}T^{-2}$, scalar, for YZ is directed parallel to X, hence $\frac{X}{YZ}$ is scalar. Couple $MXZT^{-2}$, a vector directed along Y, &c.

In deducing the dimensions of electromagnetic quantities it will be necessary to start with the dimensions of energy, or energy per unit volume. Using Cartesian units of length this is MX^2T^{-2}, MY^2T^{-2}, MZ^2T^{-2}, or MR^2T^{-2}, as the case may be; and since we shall have to deal with connected equations between the various quantities, the particular form to be used becomes of importance. Of course, energy being a scalar quantity, the above do not differ *essentially*: the difference arises only from the different ways (the different dynamical reactions) by which the expressions are derived. It might be more convenient to put $X=i\text{L}$, $Y=j\text{L}$, $Z=k\text{L}$, then the dimensions of the energy of the medium become ML^2T^{-2}. There is thus nothing to indicate how or with reference to what dynamical reactions the expression is derived. In what follows, however, Cartesian expressions will still be used though more cumbrous and involving more explanation. They may be immediately converted into the

above. The energy of the medium will be expressed in terms of an *instantaneous* linear displacement R upon which our conceptions of the electromagnetic displacements at a point, whatever their nature may be, must ultimately depend. By keeping R, at first, separate from X, Y, and Z, the formulæ gain in generality, and by retaining R, X, Y, Z instead of their equivalent vector forms rL, iL, jL, kL, the dynamical connexion between the various quantities is more clearly expressed.

The quantity π enters prominently into electromagnetic relations, and it becomes necessary at the outset to determine in what manner it is to be dealt with. This subject has been discussed by Mr. Oliver Heaviside (Elec. October 16th and 30th, 1891), and his conclusions may be briefly summarized as follows :—

If m and q be point sources of induction and displacement respectively, the measure of the induction and displacement at a distance r from the source (if the fluxes emanate isotropically) is

$$\mathbf{B}=\left(\frac{m}{4\pi r^2}\right), \quad \mathbf{D}=\left(\frac{q}{4\pi r^2}\right),$$

where $\mathbf{B}$ and $\mathbf{D}$ are the densities of the fluxes over spherical surfaces enclosing the sources. And, similarly, the density of any radial flux at a point should be estimated by the total flux through a spherical surface having its centre at the source and passing through the given point divided by the surface. Writing $\mathbf{B}=\mu\mathbf{H}$, and $\mathbf{D}=k\mathbf{E}$, where μ and k express physical properties of the medium, we have

$$\mathbf{H}=\frac{1}{\mu}\left(\frac{m}{4\pi r^2}\right), \quad \mathbf{E}=\frac{1}{k}\left(\frac{q}{4\pi r^2}\right).$$

Now, $\mathbf{H}$ and $\mathbf{E}$ express the strengths of the fields produced by the fluxes m and q at distances r from the source. Hence $\frac{m}{\mu}$ and $\frac{q}{k}$ express the strengths of the sources. Again, multiplying the above by m and q respectively we get

$$m\mathbf{H}=\frac{1}{\mu}\left(\frac{m^2}{4\pi r^2}\right), \quad q\mathbf{E}=\frac{1}{k}\left(\frac{q^2}{4\pi r^2}\right).$$

But $m\mathbf{H}$ is the force experienced by a pole m when placed in a field of strength $\mathbf{H}$, and similarly for $k\mathbf{E}$. Hence

$$F_m = m\mathbf{H} = \frac{1}{\mu}\left(\frac{m^2}{4\pi r^2}\right), \quad F_q = q\mathbf{E} = \frac{1}{k}\left(\frac{q^2}{4\pi r^2}\right),$$

where F_m and F_q are respectively the forces between two poles m, or two charges q, at distances r apart. In other words, since in expressing the force between two poles or two charges we have to regard each pole or charge as an *isolated point source of displacement*, we should regard the one pole or charge as producing radially a field of given strength, and then express the force experienced by the other when placed in this field.

If, now, the unit pole and the unit charge be defined respectively by the relations

$$m = r\sqrt{4\pi\mu F_m}, \qquad q = r\sqrt{4\pi k F_q},$$

instead of, as usual,

$$m = r\sqrt{\mu F_m}, \qquad q = r\sqrt{k F_q},$$

the effect is to *redistribute* π in electromagnetic equations as a whole. It is found, however, that all those relations into which it is now made to enter depend upon and involve the consideration of circuital or radial fluxes, and π obviously enters as a plane or solid angle in connexion with circles and spheres of reference. It has thus a definite physical meaning, and is always definitely related to the other quantities in the relation. On the other hand, in the case of the relations from which it is removed, it previously entered only because of its suppression elsewhere, and had no fundamental connexion with the other quantities.

The relation between the new and the old units thus defined are given in the papers above referred to. For the purpose of the present paper it is sufficient to notice that the relation

$$4\pi u \partial y \partial z = \left(\frac{\partial\gamma}{\partial y} - \frac{\partial\beta}{\partial z}\right)\partial y\,\partial z$$

becomes

$$u \partial y \partial z = \left(\frac{\partial\gamma}{\partial y} - \frac{\partial\beta}{\partial z}\right)\partial y \partial z :$$

$p = 4\pi m$ becomes $p = m$: $\mathbf{B} = \mu_0\mathbf{H} + 4\pi\mathbf{I}$ becomes $\mathbf{B} = \mu_0\mathbf{H} + \mathbf{I}$,

so that **B** and I are identical ; the electrostatic energy of the medium $\left(\frac{\mathbf{ED}}{4\pi}\right)$ becomes **ED**, thus harmonizing with **BH**.

Since, in these relations, π essentially preserves its primitive geometrical or trigonometrical meaning, its insertion or suppression implies the insertion or suppression of a *concrete quantity*, and not merely a number, so that the natural relations between the various quantities are affected. For this reason, therefore, the "rational units," given by Mr. Oliver Heaviside, in which π is primarily associated with the radial and circuital fluxes of the field, will alone be used in what follows.

The relations made use of in deducing the dimensions of electromagnetic quantities are of three kinds.

I. Relations between quantities of the same kind, either purely electrical or purely magnetic.—These are :—

(*a*) Electrical.

1. $\mathbf{D} = k\mathbf{E}$.
2. $C = \frac{\partial}{\partial t} . e$.
3. $e =$ Surface-integral of **D**.
4. $\mathbf{C} = \frac{\partial}{\partial t} . \mathbf{D}$.
5. $E =$ Line-integral of **E**.

Where **D**=electric displacement ; k=specific inductive capacity ; **E**=electrical force ; C=electric current ; e=quantity of electricity ; **C**=current density ; E=electromotive force or *voltage* of a closed circuit.

(*b*) Magnetic.

1. $\mathbf{B} = \mu\mathbf{H}$.
2. $C_m = \frac{\partial}{\partial t} . m$.
3. $m =$ Surface-integral of **B**.
4. $\mathbf{C}_m = \frac{\partial}{\partial t} . \mathbf{B}$.
5. $E_m =$ Line-integral of **H**.

B=magnetic induction ; μ=specific magnetic capacity ; **H**=magnetic force ; m=quantity of magnetism. Mr Oliver

Heaviside designates E_m the *Gaussage* of a closed magnetic circuit to correspond with E, the *Voltage* of the corresponding electrical one, C_m and $\mathbf{C}_m$ being the magnetic currents.

II. Relations between quantities of different kinds.—These relations are embodied in the two laws of circuitation:

$$(a) \qquad \textit{Circuitation } \mathbf{H} = C = E_m = \frac{\partial}{\partial t}.e\,;$$

or, the *Gaussage* of a closed magnetic circuit measures the total electrical current through it.

$$(b) \qquad \textit{Circuitation } \mathbf{E} = C_m = E = \frac{\partial}{\partial t}.m\,;$$

or, the *Voltage* of a closed electrical circuit measures the total magnetic current through it.

The corresponding relations as given by Maxwell are

$$(a) \qquad u\partial y\partial z = \left(\frac{\partial\gamma}{\partial y} - \frac{\partial\beta}{\partial z}\right)\partial y\partial z,$$

u being the component current-density along x, and β, γ the components of magnetic force along y and z respectively, the whole being referred to an elementary magnetic circuit $(\partial y\partial z)$. [The 4π in the expression $4\pi u$ is dropped, as previously explained.]

$$(b) \qquad P = \left(c\frac{\partial y}{\partial t} - b\frac{\partial z}{\partial t}\right),$$

neglecting $\mathbf{A}$ and ψ. P is the component electrical force along x, and b and c the components of magnetic induction along y and z respectively.

III. Dynamical Relations.—These are relations between quantities whose product express the energy, or energy per unit volume of the medium.

$eE \;:\; m\Omega \;:\; pC$ . . . Energy.

$\mathbf{DE} \;:\; \mathbf{BH} \;:\; \mathbf{AC}$ . . . Energy per unit volume.

But $\mathbf{D} = k\mathbf{E}$, and $\mathbf{B} = \mu\mathbf{H}$. Hence

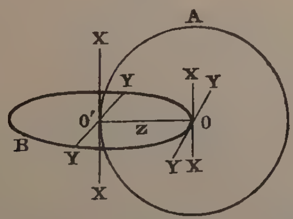
$k\mathbf{E}^2 \;:\; \mu\mathbf{H}^2$ . . Energy per unit volume.

By means of these relations we can express in terms of M, X, Y, Z, T, and *one selected quantity* the dimensions of all the rest. The only useful cases, however, are those in which the selected quantities are either μ or k, for these express

physical properties of the medium at a point, and are independent of the electromagnetic reactions going on there. The dimensions in terms of μ are obtained by starting with the relation $\mu H^2 =$ Energy per unit volume; and similarly for the dimensions in terms of k.

Since the above dimensions have to be deduced by means of a connected system of equations, it becomes necessary to make a suitable choice of axes of reference. Let X be the axis of the electric displacement, and Y that of the magnetic displacement at a point in the medium. For an isotropic medium (the only case we have at present to consider) these are mutually at right angles, and Z is at right angles to both, being the intersection of the electric and magnetic equipotential surfaces. Let this relation between the directions of the axes of reference and the displacements hold for every point of the medium, so that the axes constitute an instantaneous system at every point. In passing therefore from point to point in the medium, and for different epochs at the same point, the axes and the displacements preserve their *relative* directions, while their *absolute* direction in space in general alters.

Fig. 5.



Let AO′ be a closed electric circuit, and BO a corresponding closed magnetic circuit, both being circles in planes at right angles to each other. Taking instantaneous axes as above, every element of the circuit AO′ is ∂x, and of the circuit BO is ∂y, while an element of the intersection of the planes of the circuits is ∂z. The length of the circuit AO′ is $\Sigma\partial x$, and of the circuit BO, $\Sigma\partial y$, while the surfaces of the circuits are ultimately $\Sigma(\partial x\partial z)$, and $\Sigma(\partial y\partial z)$. We have therefore :—

1. Circuitation $\mathbf{H} = \Sigma(\mathbf{H}\partial y) = \mathrm{C}$.
2. Circuitation $\mathbf{E} = \Sigma(\mathbf{E}\partial x) = \mathrm{E}$.

3. Surface-integral of $=\mathbf{D}\Sigma(\mathbf{D}\partial y\partial z)=e$.

4. Surface-integral of $\mathbf{B}=\Sigma(\mathbf{B}\partial x\partial z)=m$.

To express these dimensionally, we have to neglect the summation Σ, and substitute for ∂x, ∂y, ∂z respectively X, Y, Z. The relations then become :—

1. $\mathbf{D} = k\mathbf{E}$.
2. $C = eT^{-1}$.
3. $e = \mathbf{D}(YZ)$.
4. $\mathbf{C} = \mathbf{D}T^{-1}$.
5. $E = \mathbf{E}(X) = C_m = mT^{-1} = \mathbf{B}(XZ)T^{-1}$.
6. $\mathbf{B} = \mu\mathbf{H}$.
7. $C_m = mT^{-1} = E$.
8. $m = \mathbf{B}(XZ)$.
9. $\mathbf{C}_m = \mathbf{B}T^{-1}$.
10. $E_m = \mathbf{H}(Y) = C = eT_{-1} = \mathbf{D}(YZ)T^{-1}$.

The energy of the medium at any point may be expressed by

$$\Sigma m(\dot{x}^2+\dot{y}^2+\dot{z}^2)=\Sigma(m\dot{r}^2),$$

where r is the instantaneous linear displacement upon which both the electric and magnetic displacements at that point depend, for the two laws of circuitation express that the electric and magnetic displacements at a point in the medium are not independent, but originate from the same dynamical cause. Expressed dimensionally, this becomes

$$M(X^2+Y^2+Z^2)T^{-2} \text{ or } MR^2T^{-2},$$

and the dimensions of energy per unit volume are

$$MR^2(XYZ)^{-1}T^{-2}.$$

Since the axes of reference (X, Y, Z) and the displacement R are both instantaneous with respect to any point, the direction of R must be definitely related to the axes, the relation being dependent upon the dynamical mechanism of the field. It will be convenient, however, to express the dimensions of electromagnetic quantities, first, in terms of M, X, Y, Z, T and R; and afterwards to determine the relation between the direction of R and the axes. The fact that the formulæ of some of the quantities involve R, while others do

not, then simply means that in the former cases the corresponding quantities depend upon and involve the instantaneous linear displacement specified by R ; while in the other cases they are independent of the displacement and express only physical properties of the medium.

I. Electromagnetic System :—

$$\mu\mathbf{H}^2 = \mathrm{MR^2T^{-2}(XYZ)^{-1}}.$$

Hence

1. $\mathbf{H} = \mu^{-\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(XYZ)^{-\frac{1}{2}}}]$.
2. $\mathbf{B} = \mu\mathbf{H} = \mu^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(XYZ)^{-\frac{1}{2}}}]$.
3. $m = p = \mathbf{B}(\mathrm{XZ}) = \mu^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})}]$.
4. $\mathrm{E} = m\mathrm{T}^{-1} = \mu^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-2}(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})}]$.
5. $\mathbf{E} = \mathrm{E(X^{-1})} = \mu^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-2}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})}]$.
6. $\mathbf{A} = \mathrm{ET} = \mu^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})}]$.
7. $\mathrm{C} = \mathbf{H}\mathrm{Y} = \mu^{-\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(X^{-\frac{1}{2}}Y^{\frac{1}{2}}Z^{-\frac{1}{2}})}]$.
8. $\mathbf{C} = \mathrm{C(YZ)^{-1}} = \mu^{-\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{3}{2}})}]$.
9. $e = \mathrm{CT} = \mu^{-\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}R(X^{-\frac{1}{2}}Y^{\frac{1}{2}}Z^{-\frac{1}{2}})}]$.
10. $\mathbf{D} = \mathbf{C}\mathrm{T} = e\mathrm{(YZ)^{-1}} = \mu^{-\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}R(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{3}{2}})}]$.

From 5 and 10 we have

$$k = \frac{\mathbf{D}}{\mathbf{E}} = \mu^{-1}[\mathrm{Z^{-2}T^2}],$$

and substituting $k^{-1}[\mathrm{Z^{-2}T^2}]$ for μ all through in the above we get for each quantity its electrostatic dimensions, the results being identical with those obtained directly as below.

II. Electrostatic System :—

$$k\mathbf{E}^2 = \mathrm{MR^2T^{-2}(XYZ)^{-1}}.$$

1. $\mathbf{E} = k^{-\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(XYZ)^{-\frac{1}{2}}}]$.
2. $\mathbf{D} = k\mathbf{E} = k^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(XYZ)^{-\frac{1}{2}}}]$.
3. $e = \mathbf{D}(\mathrm{YZ}) = k^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-1}(X^{-\frac{1}{2}}Y^{\frac{1}{2}}Z^{\frac{1}{2}})}]$.
4. $\mathrm{C} = e\mathrm{T}^{-1} = k^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-2}(X^{-\frac{1}{2}}Y^{\frac{1}{2}}Z^{\frac{1}{2}})}]$.
5. $\mathbf{C} = \mathrm{C(YZ)^{-1}} = k^{\frac{1}{2}}[\mathrm{M^{\frac{1}{2}}RT^{-2}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{3}{2}})}]$.

6. $\mathbf{H} = \mathrm{C}(\mathrm{Y}^{-1}) = k^{\frac{1}{2}}[\mathrm{M}^{\frac{1}{2}}\mathrm{R}\mathrm{T}^{-2}(\mathrm{X}^{-\frac{1}{2}}\mathrm{Y}^{-\frac{1}{2}}\mathrm{Z}^{\frac{1}{2}})]$.

7. $\mathrm{E} = \mathbf{E}(\mathrm{X}) = k^{-\frac{1}{2}}[\mathrm{M}^{\frac{1}{2}}\mathrm{R}\mathrm{T}^{-1}(\mathrm{X}^{\frac{1}{2}}\mathrm{Y}^{-\frac{1}{2}}\mathrm{Z}^{-\frac{1}{2}})]$.

8. $m = p = \mathrm{ET} = k^{-\frac{1}{2}}[\mathrm{M}^{\frac{1}{2}}\mathrm{R}(\mathrm{X}^{\frac{1}{2}}\mathrm{Y}^{-\frac{1}{2}}\mathrm{Z}^{-\frac{1}{2}})]$.

9. $\mathbf{B} = m(\mathrm{XZ})^{-1} = k^{-\frac{1}{2}}[\mathrm{M}^{\frac{1}{2}}\mathrm{R}(\mathrm{X}^{-\frac{1}{2}}\mathrm{Y}^{-\frac{1}{2}}\mathrm{Z}^{-\frac{3}{2}})]$.

10. $\mathbf{A} = \mathrm{ET} = k^{-\frac{1}{2}}[\mathrm{M}^{\frac{1}{2}}\mathrm{R}(\mathrm{X}^{-\frac{1}{2}}\mathrm{Y}^{-\frac{1}{2}}\mathrm{Z}^{-\frac{1}{2}})]$.

From 6 and 9 we have

$$\mu = \frac{\mathbf{B}}{\mathbf{H}} = k^{-1}[\mathrm{Z}^{-2}\mathrm{T}^{2}]\,;$$

and by substituting $\mu^{-1}[\mathrm{Z}^{-2}\mathrm{T}^{2}]$ all through for k in the above, we recover the electromagnetic dimensions previously deduced.

The following are examples of the manner these formulæ work out :—

1. The velocity of propagation of an electromagnetic disturbance in the medium is given by

$$\mathrm{V} = \frac{1}{\sqrt{k\mu}} = \mathrm{ZT}^{-1} \quad \text{(dimensionally)},$$

which is of the dimensions of a velocity along Z, the normal to the plane of displacements (XY).

2. The flux of energy at a point is given by

$$\mathrm{W} = (\mathbf{EH})\,;$$

and expressing $\mathbf{E}$ and $\mathbf{H}$ dimensionally in terms of μ or k we get

$$\frac{1}{\mathrm{T}}\left(\frac{\mathrm{MR}^2\mathrm{T}^{-2}}{\mathrm{XY}}\right) = \left(\frac{\mathrm{MR}^2\mathrm{T}^{-2}}{\mathrm{XYZ}}\right)\frac{\mathrm{Z}}{\mathrm{T}} = [\mathrm{W}],$$

the bracketed factor being of the dimensions of energy per unit volume, and the other a velocity in the direction Z, the axis of flux of energy.

3. The force between two poles m is given by

$$\mathrm{F}_m = (m\mathbf{H}) = \frac{1}{\mu}\left(\frac{m^2}{4\pi r^2}\right),$$

where $\mathbf{H}$ is the strength of the field produced at the one by the other. Expressed dimensionally, this becomes

$$\mathrm{F}_m = \left(\frac{\mathrm{MR}^2\mathrm{T}^{-2}}{\mathrm{Y}}\right),$$

which is of the dimensions of the space rate of variation of energy along Y, that is, the *force* along Y, as is evident from the equivalent algebraic expression

$$m\frac{\partial^2 y}{\partial t^2}=\frac{\partial}{\partial y}\Sigma(m\dot{r}^2).$$

Putting $R=rL$, and $Y=jL$, where r and j are unit vectors in the directions R and Y, and L the *scalar* unit length, we have

$$[F_m]=\frac{MR^2T^{-2}}{Y}=\frac{M(rL)^2T^{-2}}{jL}=-\frac{ML^2T^{-2}}{jL}=-\frac{MLT^{-2}}{j}$$
$$=M(jL)T^{-2}=MYT^{-2}.$$

Thus, the force between two poles is in the direction of magnetization. The reason why the force is expressed in terms of the energy of the system is that it is a *mechanical* force arising in some way from the mutual reaction between matter and the medium. The quantities m and $\mathbf{H}$ in terms of which the force between the two poles is expressed refer to the medium alone, and since nothing is known as to the relation between the medium and matter, the relation above expresses only the *resultant* reaction taking place, namely that there is a space rate of variation of the energy of the system along Y, the direction of the force. The same remarks apply to the cases below.

4. The force between two charges q is given by

$$F_q=(q\mathbf{E})=\frac{1}{k}\left(\frac{q^2}{4\pi r^2}\right),$$

or dimensionally $[F_q]=\dfrac{MR^2T^{-2}}{X}$, the space rate of variation of the energy of the system they constitute along X, the direction of the force.

5. The force per unit volume between two elements of conductors carrying currents is given by $F_c=\mathbf{BC}$, where $\mathbf{C}$ is the current-density in the one, and $\mathbf{B}$ the induction at this one due to the other. Expressed dimensionally, this becomes

$$[F_c]=\frac{MR^2T^{-2}}{XYZ^2}=\left(\frac{MRT^{-2}}{XYZ}\right)\frac{1}{Z},$$

which is of the dimensions of space rate of variation of energy

per unit volume along Z, the direction of the force, for the conductors move at right angles to their lines of force which are circles about them.

Whatever be the dimensions of μ those of k must be given by $\mu^{-1}[Z^{-2}T^2]$. Now, the ratio of the dimensions of the same quantity expressed in the two systems is a power of $\mu k[Z^2T^{-2}]$. Hence, all possible dimensional values of μ together with the corresponding ones for k will, when substituted in the formulæ, bring the two systems into accord. Of these, however, there must be one pair, and only one pair, which give rise to dimensions whose interpretations are physically real.

It is, of course, impossible to determine from purely dimensional considerations what this particular pair must be, for this would imply a knowledge of the dynamical nature of electromagnetism. It *is* possible, however, to assign to μ and k dimensions fulfilling certain *assumed* conditions. The dimensions of the remaining quantities then become unique, and we may, by deducing the physical interpretations of the formulæ, pass on to the quantities they then represent. In this way, the formulæ in terms of μ and k may be utilized as means for tracing out in detail the various analogies between electromagnetism and dynamics. For every dimensional value of μ and k we thus obtain a perfectly connected dynamical analogue of electromagnetism, which may or may not be rational and intelligible as a whole, and whose relation to physical reality depends upon the imposed conditions.

For the purpose of the present paper, let us impose upon the dimensions of μ and k the condition that the indices of the fundamental units in their formulæ are not to be higher or lower than those found in the formulæ of ordinary dynamical units. This is, of course, a *purely arbitrary condition*, and simply expresses that the dynamical analogues to be traced out are restricted to those which are of a simple, natural, and intelligible character. All other cases are purposely excluded, not, of course, as a matter of necessity, but as a matter of convenience, for nothing would be gained by the introduction of formulæ whose interpretation would be obscure and unintelligible in our present state of knowledge. Let us therefore proceed to deduce from dimensional considerations the

various analogies between electromagnetism and dynamics, *subject to the arbitrary condition imposed above.*

In the case of ordinary dynamical units we notice that

1. No fractional powers of the fundamental units occur.
2. The indices of M are never higher than ± 1.
3. " " X, Y, Z " " " ± 2.
4. " " T " " " ± 3.

This, therefore, is the range within which μ must be found subject to the above conditions.

[Of course, if the fundamental units be not L, M, T, but some of the *now* derived units, then fractional powers may occur. Thus, if V (volume) be a fundamental unit, the dimensions of length are $V^{\frac{1}{3}}$, and of area $V^{\frac{2}{3}}$, &c. So long, however, as L, M, T, are fundamental units, we cannot expect fractional powers to occur. For *length*, or space of one dimension, is the simplest conception of space which we can form, while *time* and *mass* (not necessarily that of matter, but *tangibleness* in general) are fundamental conceptions beyond which we cannot go. Now, all dynamical conceptions are built up ultimately in terms of these three ideas, mass, length, and time, and since the process is synthetical, building up the complex from the simple, it becomes expressed in conformity with the conventions of Algebra by *integral powers* of L, M, T. The analytical process, that is, splitting up a complex conception into its ultimate constituents (evolution in Algebra), becomes expressed according to the same conventions by fractional powers, *e. g.* $L=(V)^{\frac{1}{3}}$, $[\text{Area}]=(V)^{\frac{2}{3}}$, &c. But, obviously, if mass, length, and time are to be *ultimate* physical conceptions, we cannot give interpretations to fractional powers of L, M, and T, because we cannot analyse the corresponding ideas to anything simpler. We should thus be unable, according to any physical theory, to give interpretations to formulæ involving fractional powers of the fundamental units. Our only hope in such cases would be that the units themselves might cease to be fundamental. On the other hand, the building up of a derived from fundamental units is always a simple process, nothing but *whole units* of the latter kind being involved. We should thus *expect*, both from the algebraical meanings of

integral and fractional powers, and from the manner physical units are derived, to find the dimensional formulæ of physical quantities free from fractional powers. That the formulæ of all dynamical quantities, that is all the *absolute* formulæ we are acquainted with, are of this kind, is an argument in favour of this view.

A vector length has two properties, direction and magnitude. When squared, a vector length becomes scalar. A vector enters into physical relations either as a vector (having direction and magnitude) or as a scalar (having magnitude only). Hence we may say that the first and second powers of a vector, X and X^2 (say), represent the two different ways in which it enters all physical relations. For all other integral powers differ from these only in scalar magnitude. If the dimensional formulæ are to express different *natural relations* between quantities, then, so far as the unit length is concerned (the representative of space), all these relations are involved in X and X^2, X being a vector. If this be true, then the fact that the indices of vector lengths, in the formulæ of dynamical quantities, are never higher than ± 2 becomes explained. In a similar manner, we may explain the fact that the indices of M are always ± 1, for here no new idea is introduced by M^2 as in the case of vectors: it differs from M only in scalar magnitude, and still conveys the same physical idea, namely *inertia*. It is widely different in the case of time, for every new *negative* power of T introduces a new physical idea with respect to time—that is, it introduces the conception of a new *time-flux*.

It is not necessary, however, to postulate anything as to these matters. It is sufficient for our present purpose to know what the limits of the indices of the fundamental units are in the case of known dynamical quantities, without having to account for the same. The above are only suggestions.]

Let the dimensions of μ be

$$[\mu] = M^m R^r T^t (X^x, Y^y, Z^z),$$

then those of k are

$$[k] = M^{-m} R^{-r} T^{-t+2} (X^{-x} Y^{-y} Z^{-z-2}).$$

Since the indices of M, X, Y, Z in the dimensional formulæ are odd multiples of $\frac{1}{2}$, their indices in the dimensions of μ must be *odd*, otherwise, on substituting for μ and k in these formulæ their dimensional values, the formulæ would not be rationalized. For a similar reason, the indices of R and T must be *even*. Hence, the indices of M, X, Y, Z must be ± 1, and of R, 0, or ± 2. In the case of Z, $+1$ is not admissible, otherwise we should have Z^{-3} in the case of k. Thus, Z must enter both μ and k as Z^{-1}. In the case of T the possible values are $0, \pm 2, \pm 4, \ldots \pm 2n$. These values may be tabulated thus :—

$[\mu]$		$[k]$	
(a)	(b)	(a')	(b')
0	...	$+2$	...
$+2$	-2	0	$+4$
$+4$	-4	-2	$+6$
$+6$	-6	-4	$+8$
. . .	. . .	. . .	. . .
$+2n$	$-2n$	$-2(n-1)$	$+2(n+1)$

Under a are given the *positive* possible values for t in μ, under a' the corresponding values in the case of k. Under b, the *negative* values in the case of μ, and under b' those corresponding in the case of k. The only cases necessary to be considered are T^0 and T^2, for, as seen from the table, all other possible values of T lead to dimensional formulæ involving powers of T higher than ± 4.

The dimensions of μ and k involve only X, Y, Z, M and T. This is of course obvious if R has to coincide ultimately with X, Y, or Z. If R is not ultimately to coincide with X, Y, or Z, it cannot enter into the formulæ of μ and k. For R can enter into the formula of μ only as R^2 or R^{-2}. In the former case, $\mu^{\frac{1}{2}}$ contains R, and $\mu^{-\frac{1}{2}}$ contains R^{-1}. But $\mu^{\frac{1}{2}}$ is a factor of the formulæ of m, **B**, **E**, E, and $\mu^{-\frac{1}{2}}$ of e, **D**, C, **C**, **H** ; and since R is a factor of the formulæ of all the above quantities, we should have :—

i. m, **B**, **E**, E, containing R in their dimensional formulæ,

ii. e, **D**, **H**, C, **C**, containing R^0 :

thus indicating that some of the forces and fluxes of the field

would be dependent upon R, the others independent of it. But this is obviously impossible, since R is *the* instantaneous linear displacement at a point in the field upon which depend all the electromagnetic forces and fluxes at that point. In a similar way, if μ involves R^{-2}, m, **B**, **E**, E would be independent of R, and e, **D**, **H**, C, **C** dependent upon it, leading to the same conclusion as before. Thus, μ and k must be quantities independent of the electromagnetic reactions (specified by R) which may be going on in the field, as is otherwise evident since μ and k express physical properties of the medium. We have therefore to consider the different ways the dimensions of μ (say) can be built up from M, X, Y, Z, and T, subject to the conditions already laid down.

The possible dimensional values for μ are to be obtained from $M^{\pm 1} X^{\pm 1} Y^{\pm 1} Z^{-1} T^{0,+2}$, by forming all the possible combinations of the quantities taken all together. The possible cases are :—

I.	1. $MX^{-1}Y^{-1}Z^{-1}$.	II.	1. $MX^{-1}Y^{-1}Z^{-1}T^2$.
	2. $MXYZ^{-1}$.		2. $MXYZ^{-1}T^2$.
	3. $MYY^{-1}Z^{-1}$.		3. $MXY^{-1}Z^{-1}T^2$.
	4. $MX^{-1}YZ^{-1}$.		4. $MX^{-1}YZ^{-1}T^2$.
III.	1. $M^{-1}X^{-1}Y^{-1}Z^{-1}$.	IV.	1. $M^{-1}X^{-1}Y^{-1}Z^{-1}T^2$.
	2. $M^{-1}XYZ^{-1}$.		2. $M^{-1}XYZ^{-1}T^2$.
	3. $M^{-1}XY^{-1}Z^{-1}$.		3. $M^{-1}XY^{-1}Z^{-1}T^2$.
	4. $M^{-1}X^{-1}YZ^{-1}$.		4. $M^{-1}X^{-1}YZ^{-1}T^2$.

There are thus sixteen cases, and since for each value of μ we have up to the present supposed that R may ultimately coincide with X, Y, or Z, or with neither, the different systems which may be deduced from the above are sixty-four in number. There are other conditions, however, which enable us to reduce this number.

The quantities m, E, C, and e are scalar, and **B**, **E**, **C**, **D** vectors. Hence, the dimensional values of μ must be such that when substituted in the dimensional formulæ, the scalar quantities remain scalars, and the vector quantities vectors properly directed with respect to the dimensional axes. Since

all quantities are scalar so far as M and T are concerned, it is only necessary to examine how the scalar and vector character of the quantities depend upon X, Y, Z. To do this, we may take the relation $[m]=\mu^{\frac{1}{2}}[MRT^{-1}(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})]$, and substitute for μ successively the four factors $(XYZ)^{-1}$, (XYZ^{-1}), $(XY^{-1}Z^{-1})$, $(X^{-1}YZ^{-1})$. The conclusions deduced for m must obviously hold for the other quantities. Thus :—

1. Let $\mu^{\frac{1}{2}}$ contain $X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}}$. Then $[m]$ contains

$$(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}})\,R\,(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})=RY^{-1}.$$

This becomes scalar only when R coincides with Y.

2. Let $\mu^{\frac{1}{2}}$ contain $X^{\frac{1}{2}}Y^{\frac{1}{2}}Z^{-\frac{1}{2}}$. Then $[m]$ contains

$$(X^{\frac{1}{2}}Y^{\frac{1}{2}}Z^{-\frac{1}{2}})\,R\,(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})=RX.$$

This becomes scalar only when R coincides with X.

3. Let $\mu^{\frac{1}{2}}$ contain $X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}}$. Then $[m]$ contains

$$(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}})\,R\,(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})=R\times Y^{-1}.$$

This becomes scalar when R coincides with Z.

4. Let $\mu^{\frac{1}{2}}$ contain $X^{-\frac{1}{2}}Y^{\frac{1}{2}}Z^{-\frac{1}{2}}$. Then $[m]$ contains

$$(X^{-\frac{1}{2}}Y^{\frac{1}{2}}Z^{-\frac{1}{2}})\,R\,(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})=R.$$

This cannot become scalar.

Thus, in order to render the scalar quantities scalar, R must ultimately coincide with X, Y, or Z. There are thus three cases to be considered :—

1. When R coincides with Y, μ contains $(XYZ)^{-1}$.
2. „ „ „ X, „ (XYZ^{-1}).
3. „ „ „ Z, „ $(XY^{-1}Z^{-1})$.

But, according to the electromagnetic theory of light, R cannot coincide with Z, for XY is the wave-front of an instantaneous plane-polarized disturbance at a point in the medium—the disturbance ultimately originating in the displacement R, and since the medium is isotropic, the linear displacement specified by R can have no components along Z. We are thus restricted to the two cases where R coincides with Y, and the dimensions of μ involve $(XYZ)^{-1}$, or where R coincides with X, and the dimensions of μ involve XYZ^{-1}. In

the former case, the instantaneous linear displacement R of the medium at any point is in the plane of polarization (the plane of magnetic displacement), magnetic energy is kinetic, and electrical energy potential. In the latter case, the displacement R is at right angles to the plane of polarization, magnetic energy is potential, and electrical energy kinetic. Physicists are not yet agreed as to the interpretation of Weiner's results, so that no arguments as to the direction of the instantaneous displacement of the medium at a point can be based upon his experiments. We have, therefore, to suppose that R may lie along either Y or X, and the possible systems become reduced to eight, as below:—

1. $M(XYZ)^{-1}$.	2. $M^{-1}XYZ^{-1}T^2$.
3. $M(XYZ^{-1})$.	4. $M^{-1}X^{-1}Y^{-1}Z^{-1}T^2$.
5. $M(XYZ)^{-1}T^2$.	6. $M^{-1}XYZ^{-1}$.
7. $M(XYZ^{-1})T^2$.	8. $M^{-1}X^{-1}Y^{-1}Z^{-1}$.

If the dimensions of μ be (1) those of k are (2), and if those of k are (1), the dimensions of μ are (2), and similarly for the other three pairs.

The dimensional formulæ 5 and 7 are unintelligible, for we have no dynamical units involving positive powers of both M *and* T, and it is difficult to give to such formulæ an unintelligible interpretation. The same difficulty appears in 6 and 8, for if one of these be the dimensional formula for μ, the corresponding one for k is 5 or 7. Apart, however, from this difficulty, these formulæ 5, 6, 7, and 8 lead to fluid theories of electromagnetism. Thus, from 5, we get for the dimensions of m:—

$$[m]=\mu^{\frac{1}{2}}[M^{\frac{1}{2}}RT^{-1}(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})]=M^{\frac{1}{2}}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}})R(X^{\frac{1}{2}}Y^{-\frac{1}{2}}Z^{\frac{1}{2}})$$
$$M^{\frac{1}{2}}T^{-\frac{1}{2}}T^{\frac{1}{2}}=MRY^{-1}=M \text{ (since R here must coincide with Y).}$$

Similarly, from 7 we have $[m]=MX^2$. Again, from 6 we get $[e]=M$, and from 8, $[e]=MY^2$. Thus, we have to suppose m to be a quantity of the nature of *mass* or *moment of inertia*, and similarly for e. In both cases, m and e would be *properties* of the medium depending upon its inertia, instead of being parts of the electromagnetic reactions going on in the

medium,—an electrical current would thus be a quantity of the nature of momentum, which is inconsistent with Maxwell's theory of electromagnetism. Other and not less serious difficulties will be met with if an attempt is made to develop their interpretations more fully.

Similar remarks apply to the systems arising from 3 and 4. Thus, from 3 we have

$$[e]=\mu^{-\frac{1}{2}}[\mathrm{M}^{\frac{1}{2}}\mathrm{R}(\mathrm{X}^{-\frac{1}{2}}\mathrm{Y}^{\frac{1}{2}}\mathrm{Z}^{-\frac{1}{2}})]$$
$$=\mathrm{M}^{-\frac{1}{2}}\mathrm{X}^{-\frac{1}{2}}\mathrm{Y}^{-\frac{1}{2}}\mathrm{Z}^{\frac{1}{2}}\mathrm{M}^{\frac{1}{2}}\mathrm{R}\mathrm{X}^{-\frac{1}{2}}\mathrm{Y}^{\frac{1}{2}}\mathrm{Z}^{-\frac{1}{2}}=\mathrm{R}\mathrm{X}^{-1}=1,$$

since R here coincides with X. Thus, e is of the dimensions of a *volume-strain*, the ratio of two identical concretes. Similarly, we get from 4, $[m]=1$, also a volume-strain. Again, in the former case we have $\mathbf{D}=e(\mathrm{YZ})^{-1}$ =volume-strain per unit area. The two currents C and **C** become volume-strain per unit time (T^{-1}, or $\mathrm{Y}^{-1}\mathrm{Z}^{-1}\mathrm{T}^{-1}$). Electrical force **E** becomes MXT^{-2}, and voltage E $=\mathrm{MX}^2\mathrm{T}^{-2}$ (energy). Since $m=\mathrm{ET}$, m becomes $\mathrm{MX}^2\mathrm{T}^{-1}$, and magnetic moment $ml = \mathrm{MX}^2\mathrm{YT}^{-1}$. Again, **B** magnetic induction becomes $m(\mathrm{XZ})^{-1} = \mathrm{MXZ}^{-1}\mathrm{T}^{-1}$, and **H** magnetic force ($\mathrm{Y}^{-1}\mathrm{T}^{-1}$). Again, in the second case, we have $\mathbf{B}=m(\mathrm{XZ})^{-1}$=volume-strain per unit area; $\mathrm{E}=\mathrm{MT}^{-1}$ $=\mathrm{T}^{-1}$=volume-strain per unit time; $e=\mathrm{MY}^2\mathrm{T}^{-1}$; $\mathrm{C}=$ $\mathrm{MY}^2\mathrm{T}^{-2}$ (energy) and **C** (current density)$=\mathrm{MY}^2(\mathrm{YZ})^{-1}\mathrm{T}^{-2}$. These formulæ, although it may be possible to give them interpretations, are at present unintelligible, and do not suggest any connected relation between the various quantities.

The cause of the unintelligible character of these latter formulæ lies in the fact that the formula MXYZ^{-1} is not symmetrical with respect to the dimensional axes X, Y, Z. Since the above formula is independent of T, the corresponding quantity must be of the dimensions of some property of the medium depending ultimately upon the *inertia* of the medium alone. It is difficult, however, to conceive what property depending upon the inertia of the medium can involve the *three* dimensional axes unsymmetrically except the rotational inertia, or moment of inertia of a unit volume. The above formula, however, does not admit of such an interpretation. Thus, whether MXYZ^{-1} be the dimensional formula of μ or k,

the results are either incapable of interpretation (as, for example, μ and k), or the interpretations are unintelligible.

Thus there are left only cases 1 and 2. If the dimensions of μ be those of 1, that is $M(XYZ)^{-1}$, μ becomes the inertia of unit volume, or the density of the medium, and magnetic energy is kinetic. If the dimensions of μ be 2, those of k are 1, and k becomes the density of the medium, and electrical energy is kinetic. There are thus two cases to be discussed.

I. *Magnetic Energy Kinetic.*—μ the density of the medium.

1. Magnetic force $=\mathbf{H}=\mu^{-\frac{1}{2}}[M^{\frac{1}{2}}RT^{-1}(XYZ)^{-\frac{1}{2}}]=RT^{-1}=YT^{-1}$, since R must now coincide with Y. This is the linear velocity directed along Y the magnetic axis.

2. Magnetic induction $=\mathbf{B}=$ Intensity of magnetization, $\mathbf{I}=\mu\,\mathbf{H}=\left(\frac{M}{XYZ}\right)\frac{Y}{T}=$ Linear momentum per unit volume.

3. Magnetic moment $=ml=\mathbf{I}\,(XYZ)=MYT^{-1}=$ Linear momentum.

4. Current strength $=C=\mathbf{H}Y=Y^2T^{-1}$.

5. Current density $=\mathbf{C}=C\,(YZ)^{-1}=YZ^{-1}T^{-1}=$ Angular velocity. The velocity is instantaneously directed along Y, the magnetic axis, and the axis of rotation is X, the axis of the electrical displacement. If the angular velocity be that of a vortex filament coinciding with the current, the strength of the current, C, becomes the strength of the vortex—product of angular velocity into cross-section of filament.

6. Electric displacement $=\mathbf{D}=CT=YZ^{-1}=$ an angular displacement. [According to the elastic solid theory, $\mathbf{D}$ would be of the dimensions of a *shear*, the planes XY being displaced parallel to each other in the direction Y.]

7. Electrical force $=\mathbf{E}$. Since $[\mathbf{ED}]=MY(XZ)^{-1}T^{-2}$, and $[\mathbf{D}]=YZ^{-1}$, $\mathbf{E}=MX^{-1}T^{-2}$. Taking $\mathbf{D}$ as an angular displacement, $\mathbf{E}$ must be of the dimensions of a *torque*, for $\mathbf{ED}$ is of the dimensions of work done per unit volume. [Let a cylinder rotate in a resisting medium. The resistance to the rotation is a tangential force per unit surface of the cylinder, directed everywhere parallel to the motion. If the axis of the cylinder be X (the electrical axis), and the radius Z, an element of its surface is of the dimensions YZ. Hence, the

resisting stress is of the dimensions $MX^{-1}T^{-2}=[\mathbf{E}]$. Thus $\mathbf{E}$ is a tangential force per unit area. Now $\mathbf{E}$ has a moment round X. Hence we have

$$[\mathbf{E}]=\left(\frac{MYZT^{-2}}{XYZ}\right),$$

indicating that the resisting stress over an element of surface is inversely as the distance of the element from the axis of rotation, and its moment directly as that distance.]

8. Voltage.—Writing $[\mathbf{E}]=\frac{MYZT^{-2}}{XYZ}$, we get

$$[E]=\left(\frac{MYZT^{-2}}{YZ}\right).$$

Now E, the line integral of $\mathbf{E}$, is the source of the disturbance in a closed electrical circuit. Hence, taking the above mechanical illustration, E becomes the terminal torque required to maintain the motion of the cylinder against the resistance. "The terminal torque corresponds to the impressed voltage. It should be so distributed over the end B" (in this case a rotating tube) "that the applied force there is a circular tangential traction varying inversely as the distance from the axis" (Mr. Oliver Heaviside, Elec. Jan. 23, 1891). In the above case, an element of the tangent at a point is Y, and of the radius Z.

9. Vector potential $=\mathbf{A}=\mathbf{E}T=MX^{-1}T^{-1}=\frac{MYZT^{-1}}{XYZ}$

= angular momentum per unit volume.

10. Specific resistance.

$$\rho=\frac{\mathbf{E}}{\mathbf{C}}=\frac{MX^{-1}T^{-2}}{YZ^{-1}T^{-1}}=MZ(XY)^{-1}T^{-1}$$

= coefficient of viscosity = tangential force per unit area ÷ shear per unit time.

11. Self-induction.—Let a linear conductor carry a current C. Its self-induction is given by $L=\frac{p}{C}$, or dimensionally MY^{-2}. If $\mathbf{L}$ be defined as the self-induction of the conductor for a current of unit density,

$$\mathbf{L}=\frac{p}{\mathbf{C}}=MT^{-1}(YZ^{-1}T^{-1})^{-1}=MZY^{-1}=\frac{MZ}{YZ}$$

= moment of inertia per unit area; or $= \left(\frac{M}{YZ}\right)Z^2$ = moment of inertia of a disk of unit mass per unit area.

12. Specific inductive capacity $= k = [MZ(XY)^{-1}T^{-2}]^{-1}$. On the elastic solid theory k^{-1} would be defined as the rigidity of the medium. But $k^{-1} = RT^{-1}$, where R is specific resistance. Thus the interpretations of k and R go together. If R (as above) be a coefficient of viscosity, k^{-1} becomes "a *quasi* rigidity" of the medium "arising from elastic resistance to absolute rotation." (Mr. Oliver Heaviside, Elec. Jan. 23, 1891.)

13. Electrical charge $= q = \mathbf{D}(YZ) = Y^2 =$ (Product of angular displacement into area).

14. Magnetic pole $= m = \frac{ml}{l}$ = Magnetic moment per unit length = Linear momentum per unit length.

The connexion between Vortex Motion and Electromagnetism is shown in Basset's 'Hydrodynamics,' from which the following are extracted (vol. i. chap. iv.):—

1. Vortex filament = Electrical current.
2. Velocity of liquid (linear velocity, components u, v, w) = Magnetic force (components α, β, γ).
3. Molecular rotation (angular velocity, components ξ, η, ζ) = Current density (components u, v, w).
4. Velocity potential due to vortex (ϕ) = Magnetic potential of current (Ω).
5. Doublet sheet = Magnetic shell.
6. Circulation (k) = Work done in moving a magnetic pole once round current.
7. Flux through vortex = Potential energy of magnetic shell.
8. The action of a vortex filament upon the surrounding liquid is determined by the quantities L, M, N. These correspond to the components F, G, H of electromagnetic momentum ($\mathbf{A}$).

Thus, when μ is of the dimensions of density and magnetic energy kinetic, the interpretations of the dimensional formulæ of electromagnetic quantities are identical with those of the corresponding quantities in the case of vortex motion.

In the Phil. Mag. vol. xxxi. p. 149, Prof. J. J. Thomson develops a method of representing electromagnetic effects by

means of tubes of electrostatic induction distributed throughout the field. The axes of these tubes coincide with the axes of electric displacement, and the tubes terminate normally upon the surfaces of charged bodies. By means of these tubes it is possible to picture the changes going on when electricity passes through electrolytes and conductors, and when changes are produced in the electromagnetic field. If, now, we suppose these tubes of electrostatic induction to be vortex filaments, this method of representing electromagnetic phenomena becomes identical with that indicated by the interpretations above given to the dimensional formulæ.

The two rotational theories of electromagnetism which have been hitherto put forward are discussed by Mr. Oliver Heaviside (Elec. Jan. 23, 1891). The first of these is that already deduced above from dimensional considerations, by supposing μ to be of the dimensions of density. Here the axis of rotation at any point is the axis of the electric displacement; the angular velocity is the current-density; the velocity is magnetic force; the *torque* called into play when the angular velocity of a vortex changes is the electric force; the terminal torque, the source of the disturbance, is the voltage; the relative angular displacements of the vortices when their velocities change is the electric displacement; the linear momentum of the irrotationally moving liquid is magnetic induction or intensity of magnetization; the angular momentum is the vector potential; the strength of a vortex is the strength of the current; specific resistance is a quantity of the nature of viscosity, and k^{-1} of the nature of rigidity.

In the other system things are inverted. Here k is the density of the medium, electrical energy is kinetic, and the axes of the vortices coincide with the directions of the magnetic displacements. The interpretations of the formulæ of the various quantities are as below.

II. *Electrical Energy Kinetic.*—k the density of the medium.

1. $$\mathbf{E}=k^{-\frac{1}{2}}[M^{\frac{1}{2}}RT^{-1}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}})]$$
$$=M^{-\frac{1}{2}}(XYZ)^{\frac{1}{2}}M^{\frac{1}{2}}RT^{-1}(X^{-\frac{1}{2}}Y^{-\frac{1}{2}}Z^{-\frac{1}{2}})=RT^{-1}=XT^{-1}$$
(since R here must coincide with X).

Thus, **E** is the velocity of the medium.

2. $\mathbf{E}=\mathbf{E}X=X^2T^{-1}$. Magnetic current. (See below, its interpretation depends upon that of m and $\mathbf{B}$.)

3. $m=ET=X^2$.

4. $\mathbf{B}=m(XZ)^{-1}=XZ^{-1}=$ an angular displacement. Thus, $m=$ product of angular displacement into cross section.

5. $\mathbf{C}_m=\mathbf{B}T^{-1}XZ^{-1}T^{-1}=$ Angular velocity. Thus, E the strength of the magnetic current corresponds to the strength of a vortex, $\mathbf{C}_m$ being the density of the magnetic current.

6. $e=MT^{-1}$. This corresponds to magnetic pole in the previous case.

7. $\mathbf{D}=e(YZ)^{-1}=M(YZ)^{-1}T^{-1}=\dfrac{MXT^{-1}}{XYZ}=$ Linear momentum per unit volume.

8. $\mathbf{C}=\mathbf{D}T^{-1}=\left(\dfrac{MXT^{-2}}{XYZ}\right)=$ Force per unit volume.

9. $C=MT^{-2}$. This corresponds to *voltage* in the previous case.

10. $\mathbf{H}=C(Y^{-1})=MY^{-1}T^{-2}$. Tangential force per unit area, a quantity of the nature of a torque, corresponding to $\mathbf{E}$ in the former case.

It is thus seen that this system is simply the inverse of the other, and it is unnecessary, therefore, to go into more detail.

To summarize, therefore, we may say that, of the eight possible systems previously mentioned, only two give rise to dimensions whose interpretations are intelligible, natural, and connected as a whole; and these interpretations accord with the two rotational theories of electromagnetism which have been hitherto put forward. Of the other six, it was shown that four necessitated, in some form or other, fluid theories of electricity or magnetism, and that the interpretation of the formulæ of μ and k and other quantities are difficult. These remarks also apply to the other two cases, in which electrification and magnetization became respectively *volume-strains*. All other cases were disposed of by excluding :—

(1) All values of μ leading to dimensions involving higher or lower (fractional) powers of the fundamental units than those encountered in the case of ordinary dynamical quantities.

(2) All values which while satisfying (1) rendered scalar magnitudes vectors, or vectors scalar.

(3) All values which while satisfying (2) rendered the direction of the instantaneous linear displacement at a point in an isotropic medium parallel to Z, and therefore normal to the plane of displacement (XY).

In the foregoing discussion I have attempted to generalize the ordinary dimensional expressions for physical quantities, and to carry somewhat further than he did the suggestion of Prof. S. P. Thompson that the idea of direction may be associated with the symbols as well as that of numerical magnitude. This is clearly possible—as the examples given show—in the case of ordinary dynamical quantities, and the method certainly enables us to distinguish between things which are physically different, though (if the ordinary system in which the idea of direction is suppressed be used) the dimensions are the same. The dimensional formula thus becomes for a physical quantity the analogue of a structural formula for a chemical compound.

Applying these ideas to the more difficult case of electrical and magnetic quantities, with the actual nature of which we are imperfectly acquainted, I have tried to use the method as an instrument for discriminating between probable and improbable hypotheses. It is at all events interesting to note that we are thus led to *two* possible dimensional systems which agree with the two principal groups of analogies by which electrical and magnetic facts have been illustrated by those who have most deeply studied these from the dynamical point of view. Between these two I do not attempt to decide; but I cannot but hope that the methods I have suggested may make the study of dimensional formulæ not merely a convenient method of expressing numerical relations between fundamental and derived units, but a means of seeing more deeply into the physical facts they represent.

NOTE.—I regret that when I communicated the above paper to the Physical Society I was unaware of an important article by Prof. A. Lodge on "The Multiplication and Division of Concrete Quantities" ('Nature,' July 19, 1888). In this article Prof. Lodge discusses the general question of the products and quotients of concrete quantities, and of the

meaning of physical relations between concretes of different kinds. He clearly points out that the dimensions of quantities do not always afford a test of their identity, and that in particular concretes of the order of angles and angular displacements are treated as pure numbers. The present paper may be regarded as an attempt to remove such difficulties by taking as fundamental in our theory of dimensions the vector instead of the cartesian unit length. By expressing the formulæ according to the conventions of vector algebra, we thus assign dimensions (in the extended meaning of the term) to all quantities which are physically recognized as *concrete*, the only quantities having unity as their dimensional formula being pure numbers and quantities of the nature of *tensors*, that is, ratios between similar and similarly directed concretes. Since, as pointed out by Prof. Lodge, the cause of the above difficulties lies in the neglect of the directional relations of quantities, these modified formulæ approximate more closely to what they are sometimes, and conveniently taken to be, namely *conventional expressions* of the *kinds* of different physical quantities.

INDEX.

A.

B.

C.

D.

E.

F.

G.

H.

I.

K.

L.

M.

N.

O.

P.

R.

S.

Y

Z.

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1890-91.

February 7th, 1890.

Prof. A. W. Reinold, Past President, in the Chair.

The following were elected Members of the Society :—

Mr. E. W. Smith and Mr. C. E. Holland, B.A.

The following communication was completed :—

"On Galvanometers." By Prof. W. E. Ayrton, F.R.S., Mr. T. Mather, and Dr. W. E. Sumpner.

February 21st, 1890.

Prof. G. Carey Foster, Past President, in the Chair.

Mr. S. Evershed was elected a Member of the Society.

The following communications were made :—

"On a Carbon Deposit in a Blake Telephone Transmitter." By Mr. F. B. Hawes.

"The Geometrical Construction of Direct-reading Scales for Reflecting Galvanometers." By Mr. A. P. Trotter.

"A Parallel Motion suitable for Recording Instruments." By Mr. A. P. Trotter.

March 7th, 1890.

Prof. W. E. Ayrton, President, in the Chair.

The following were elected Members of the Society:—

Sir H. Mance, Mr. L. R. Shorter, Mr. C. Thompson, and Mr. A. D. Waller.

The following communications were made:—

"On Bertrand's Refractometer." By Prof. S. P. Thompson.

"An Apparatus for Distilling Mercury in a Vacuum." By Prof. Dunstan and Mr. Dymond.

"The Theory of Osmotic Pressure and its bearing on the Nature of Solution." By Mr. S. U. Pickering, M.A.

March 21st, 1890.

Prof. W. E. Ayrton, President, in the Chair.

Mr. A. E. Childs was elected a Member of the Society.

The following communications were made:—

"The Villari Critical Point in Nickel." By Mr. Herbert Tomlinson, F.R.S.

"On Bertrand's Idiocyclophanous Prism." By Prof. S. P. Thompson.

"On the Shape of Moveable Coils used in Electrical Measuring Instruments." By Mr. T. Mather.

April 18th, 1890.

Prof. W. E. AYRTON, President, in the Chair.

Mr. W. B. CROFT was elected a Member of the Society.

The following communications were made :—

"On same recent Magnetic Work." By Prof. RÜCKER, F.R.S.

"A Theory of Permanent Magnetism." By M. M. F. OSMOND.

May 2nd, 1890.

Prof. W. E. AYRTON, President, in the Chair.

The following communications were made :—

"The Distribution of Flow in a Strained Elastic Solid." By Prof. C. A. CARUS-WILSON.

"On Photographs of Rapidly Moving Objects." By Prof. C. V. BOYS, F.R.S.

"On the Oscillating Electric Spark." By Prof. C. V. BOYS, F.R.S.

May 16th, 1890.

Prof. W. E. AYRTON, President, in the Chair.

Professor W. COLEMAN was elected a Member of the Society.

The following communications were made :—

"On Huyghens' Gearing in Illustration of Electric Induction." By Lord RAYLEIGH.

"On Dr. König's Researches on the Physical Basis of Music." By Prof. S. P. THOMPSON.

June 6th, 1890.

Prof. W. E. AYRTON, President, in the Chair.

The following communications were made :—

"The Effect of Change of Temperature on the Villari Critical Point of Iron." By Mr. H. TOMLINSON, F.R.S.

"On the Diurnal Variation of the Magnet at Kew." By Mr. W. G. ROBSON and Mr. S. W. J. SMITH.

June 20th, 1890.

Prof. W. E. Ayrton, President, in the Chair.

The following communications were made :—

"On the Stretching of Liquids." By Professor A. W. Worthington.

"On the Measurement of Electromagnetic Radiation." By Mr. C. V. Boys, Mr. A. E. Briscoe, and Mr. W. Watson.

"Notes on Secondary Batteries." By Dr. Gladstone, F.R.S., and Mr. Hibbert.

"An easy Rule for Calculating approximately the Self-Induction of a Coil." By Prof. J. Perry, F.R.S.

November 14th, 1890.

Prof. W. E. Ayrton, President, in the Chair.

The following communications were made :—

"On certain Relations existing among the Refractive Indices of the Chemical Elements." By the Rev. T. Pelham Dale, M.A.

"On Tables of Spherical Harmonics, with Examples of their Practical Use." By Prof. J. Perry, F.R.S.

November 28th, 1890.

Prof. W. E. Ayrton, President, in the Chair.

The following communications were made :—

"Additional Notes on Secondary Batteries." By Dr. Gladstone, F.R.S., and Mr. Hibbert.

"An Illustration of Ewing's Theory of Magnetism." By Prof. S. P. Thompson.

"The Solution of a Geometrical Problem in Magnetism." By Mr. T. H. Blakesley, M.A.

December 12th, 1890.

Prof. W. E. AYRTON, President, in the Chair.

The following communications were made :—

"Some Experiments with Selenium Cells." By Mr. SHELFORD BIDWELL, F.R.S.

"On Alternate Current Condensers." By Mr. JAMES SWINBURNE.

January 16th, 1891.

Prof. W. E. AYRTON, President, in the Chair.

The following communications were made :—

"On Photo-Electricity." By Prof. G. M. MINCHIN, M.A.

"A Lecture-Room Apparatus for determining the Acceleration due to Gravity." By Prof. F. R. BARRELL, B.A.

Annual General Meeting.

February 13th, 1891.

Prof. A. W. REINOLD, Past President, in the Chair.

The following Report of the Council was read by the Chairman :—

The Council have this year, as heretofore, to report a steady increase in the number of Members of the Society, which now reaches 365.

It is hoped that a perseverance in the present day and hour of Meeting, viz. *Alternate Fridays*, at 5 P.M., will ultimately result in a larger attendance at the ordinary Meetings. Since the last Annual General Meeting there have been 13 ordinary Meetings, the average attendance being 45. On one occasion, the 16th of May, when Prof. S. P. Thompson and M. König displayed to the Society the very beautiful Acoustic apparatus which the latter gentleman has carried to extreme perfection, the meeting was a thoroughly crowded one.

The translation of Van der Waals's memoir on the Continuity of the Liquid and Gaseous States of Matter, by Prof. Threlfall, Member of the Society, and Mr. J. F. Adair, has been completed and issued to Members of the Society.

Volta's Works.—The Council hope to issue the translation of Volta's works before the next Annual Meeting.

The loss of Members by death has been, the Council is glad to say, very small this year—Prof. Lant Carpenter being the only Member of the Society whose name occurs in the Obituary notice of the 'Times' at the end of 1890.

No loss has occurred in the ranks of the Honorary Members of the Society.

The following additions have been made to the Library :—

Newspapers and Magazines :—

Nature.
The Electrical Review.
Engineering.
The Electrician.
The Electrical Engineer.
Electrical Plant.
The Open Court.
Crónica Cientifica.
Ingeniero y Ferretero Español y Sud Americano.
The Philosophical Magazine.
Beiblätter der Physik.
Annalen der Physik.
Journal de Physique.

Journals of Societies, British :—

Proceedings of the Royal Society.
Journal of the Society of Arts.
Journal of the Institute of Electrical Engineers.
Proceedings of the Institute of Mechanical Engineers.
Quarterly Journal of the Royal Meteorological Society.
Proceedings and Transactions of the Royal Dublin Society.
Transactions of the Middlesex Natural History and Science Society.
Proceedings of the Philosophical Society of Glasgow.
Proceedings of the Philosophical Society of Cambridge.
Transactions of the Philosophical Society of Cambridge.

Memoirs of the Manchester Literary and Philosophical Society.
Proceedings of the Royal Institution.
Transactions of the Geological Society of Glasgow.
Proceedings of the Birmingham Philosophical Society.

Journals of Societies, Colonial :—

Journal and Proceedings of the Royal Society of New South Wales.
Proceedings of the Canadian Institute.
Transactions of the New Zealand Institute.
Magnetic and Meteorological Observations, Bombay.
Proceedings and Transactions of the Nova Scotian Institute of Natural Science.

Journals of Societies, Foreign :—

Proceedings of the Academy of Natural Sciences, Philadelphia.
Journal of the Physical and Chemical Society of Russia.
Proceedings of the Physical Society of Berlin.
Proceedings of the Physical Society of France.
Journal of the College of Science of Japan.
Transactions of the Seismological Society of Japan.

Books, &c. :—

Presented by the Astronomical Society :

Cavallo's Elements. 4 vols. 1803.
Electricity. Adams. 1784.
Electricity. Watson. 1746.
Electricity. Bennet. 1789.
Electricity. Ferguson. 1770. And a duplicate.
Electro-Chemistry. Singer. 1814.
Electricity. Priestly. 1778 & 1767.
Electricity. Cavallo. 2 vols. 1795.
Magnetism. Cavallo. 1795.
Galvanism. Wilkinson. 2 vols. 1804.
Electricity. B. Franklin. 1760, 1769.
Air-pump and Barometer. Brook. 1789.
Galvanism. Aldini. 1803.
Works of B. Franklin. 3 vols.
Natural Philosophy. Muschenbroek. 2 vols. 1744.
Magneto-Electric Light. Holmes.
Philosophical papers. B. Franklin. 1787.
Electricity. Viscount Mahon. 1779.
Magnetism and the Magnetic Needle. Lorimer. 1795.

Annual Report of Geological and Natural History Survey of Canada.

Report of the Ohio Meteorological Bureau.

On the Condensation of the Vapour of Water.

Catalogue of Books, Radcliffe Library.

Greenwich Magnetic and Meteorological Observations.

Greenwich Spectroscopic and Photographic Results.

Norwegian North-Sea Expedition. Actinida.

Dioptric Formulæ of Cylindrical Lenses. A Metric System of measuring and numbering Prisms and Lenses. Both by C. F. Prentice. New York.

Smithsonian Report, 1886 & 1887.

Annual Report of the Michigan Board of Agriculture.

A thick Octavo in Russian.

Decimal Coinage, Weights, and Measures.

On the Mathematical Theory of Light, by de Colnat d'Huart.

List of Canadian Hepaticæ.

Catalogue of Canadian Plants. Acrogens.

Scientific Papers of Clerk Maxwell. 2 vols.

Report of the Physical and Chemical Society of Russia on the Total Eclipse of the Sun of 1887.

Laplace's Works. Vol. 8.

On the Kerr Magnetic Effect, by Dr. R. Sissingh.

Abridgments of Specifications of Patent Office. 5 vols. By W. H Walenn.

W. H. SNELL, editor of the 'Electrician' newspaper, was born in the year 1858, and, dying on the 5th of March 1890, had but reached the early age of 31 when his friends were called upon to deplore his untimely end. Though he had been but a short time in the work which suited him so well, he certainly had made considerable progress in the useful but oftentimes desperate problem of making technical journalism scientific. Careful study of scientific literature and considerable experience in electrical work forbade him to allow dogma to outweigh science, and his sagacity enabled him to correlate unusual phenomena with the requirements of long and well established scientific principles.

The usefulness of such a mind would have been felt probably in any calling, but when, in 1886, the proprietors of the 'Electrician'

newspaper appointed him Editor, he had his especial opportunity, and his efforts were steadily directed into converting that journal into a trustworthy record of the progress taking place in the branch of Physics with which it purported to deal. He became Member of this Society, April 28th, 1888.

Mr. WILLIAM LANT CARPENTER was born in the year 1841, and was a son of the distinguished physiologist, Dr. W. B. Carpenter, whom he accompanied in the exploration voyages of H.M.S. 'Porcupine' in the years 1869–70, being in charge of some of the chemical and physical investigations of that Expedition. For many years he was engaged in a manufacturing business at Bristol, where he took a part in the movement for promoting the foundation of the University College of that town. In the course of his travels he did his best by means of lectures to render the study of science popular in many quarters of the British Empire, and, as Gilchrist Lecturer, at home. In his latter years he was one of the managers of the School of Electrical Engineering in Hanover Square. He died, working hard at this post, on the 23rd of December, 1890.

He was also an active supporter of the Morley Memorial College, and of the movement for the Extension of University Teaching. He was elected a Member of this Society January 28th, 1882, and cooperated with our late President, Prof. Balfour Stewart, in some of his investigations on terrestrial magnetism.

The adoption of the Report was proposed and carried unanimously.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows :—

President.—Prof. W. E. AYRTON, F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. GLADSTONE, F.R.S. ; Prof. G. C. FOSTER, F.R.S. ; Prof. W. G. ADAMS, M.A., F.R.S. ; Sir WM. THOMSON, D.C.L., LL.D., P.R.S. ; Prof. R. B. CLIFTON, M.A., F.R.S. ; Prof. A. W. REINOLD, M.A., F.R.S.

Vice-Presidents.—Dr. E. ATKINSON ; WALTER BAILY, M.A. ; Prof. O. J. LODGE, D.Sc., F.R.S. ; Prof. S. P. THOMPSON, D.Sc.

Secretaries.—Prof. J. PERRY, D.Sc., F.R.S. ; T. H. BLAKESLEY, M.A., M.I.C.E.

Treasurer.—Prof. A. W. Rücker, M.A., F.R.S.

Demonstrator and Librarian.—C. Vernon Boys, F.R.S.

Other Members of Council.—Shelford Bidwell, M.A., LL.B., F.R.S.; W. H. Coffin; Major-General E. R. Festing, R.E., F.R.S.; Prof. G. F. Fitzgerald, M.A., F.R.S.; Prof. J. V. Jones, M.A.; Rev. F. J. Smith, M.A.; Prof. W. Stroud, D.Sc.; H. Tomlinson, B.A., F.R.S.; G. M. Whipple, B.Sc.; James Wimshurst.

Votes of thanks were passed to the Lords Committee of the Council on Education; to the Officers; and to the Auditors.

The Treasurer in Account with the Physical Society, from January 1st, 1890, to December 31st, 1890.

Dr.	£	s.	d.	£	s.	d.
Balance in Bank, January 1, 1890				211	14	0
Entrance-Fees	15	0	0			
Subscriptions for 1887	1	0	0			
„ 1888	4	0	0			
„ 1889	16	0	0			
„ 1890	127	0	0			
„ 1891	2	0	0			
Excess Subscriptions		4	0			
Ten Life Subscriptions	100	0	0			
				265	4	0
One year's Dividend on £400 Furness 4 per cent. Debenture Stock, less Income Tax	15	12	0			
One year's Dividend on £575 Midland 4 per cent. Preference Stock, less Income Tax	22	8	6			
One year's Dividend on £200 Lancaster Corporation Stock, less Income Tax	7	16	0			
Three Quarters of a year's Dividend on £200 Metropolitan Board of Works Stock, less Income Tax	5	2	6			
One year's Dividend on £254$\frac{2}{5}$ New South Wales 3½ per cent. Stock	8	13	6			
				59	12	6
Received from Taylor and Francis:—						
Sales in 1890				50	13	0
Balance due to Treasurer				9	18	10
				£597	2	4

Cr.	£	s.	d.	£	s.	d.
Balance due to Treasurer				7	5	6
Excess Subscriptions returned					4	0
Taylor and Francis:—						
Proceedings, vol. x. (parts 2 & 3)	105	17	0			
Circulars, addressing and posting	7	17	0			
Members' separate copies	19	4	0			
Miscellaneous printing	25	9	0			
Physical Memoirs	72	9	9			
				230	16	9
H. and W. Brown				1	3	9
Dr. Lawson				8	10	8
Williams and Norgate				3	9	0
Haggard (Binding)				4	14	7
Gyde (Binding)				1	18	0
Attendance at Meetings and incidental expenses				7	10	3
Reporting				19	6	8
Bank charges					4	0
Petty Cash:—						
Professor Rücker				1	0	0
Professor Perry					19	4
Balance in Bank				309	19	10
				£597	2	4

Audited and found correct,

GEORGE FULLER,
A. H. FISON, } *Auditors.*

February 10th, 1891.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, December 31, 1890.

Assets.	£	s.	d.	Liabilities.	£	s.	d.
Balance in Bank	309	19	10	Balance due to Treasurer	9	18	10
Subscriptions due	194	0	0	Subscriptions anticipated	4	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0				
£575 Midland 4 per cent. Preference Stock	650	0	0				
£200 Metropolitan Board of Works Stock	217	0	0				
£200 Lancaster Corporation Stock	224	0	0				
£254¾ New South Wales 3½ per cent. Stock	249	7	5				
One Quarter's Dividend on £200 Metropolitan Board of Works Stock	1	14	2	Balance	2288	2	7
	£2302	1	5		£2302	1	5

Audited and found correct,

A. H. FISON,
GEORGE FULLER, } *Auditors.*

February 10th, 1891.

PROCEEDINGS

AT THE

MEETINGS OF THE PHYSICAL SOCIETY

OF LONDON.

SESSION 1891–92.

February 13th, 1891.

Prof. A. W. REINOLD, Past President, in the Chair.

The following were elected Members of the Society:—

Mr. W. THORP, Mr. G. W. YULE, and Mr. S. JOYCE.

The following communication was made:—

"On the Change in the Absorption-Spectrum of Cobalt Glass produced by Heat." By Sir JOHN CONROY, Bart., M.A.

Prof. MINCHIN showed some experiments in illustration of his paper on "Photo-electricity."

February 27th, 1891.

Prof. W. E. AYRTON, President, in the Chair.

Prof. A. GRAY, M.A., was elected a Member of the Society.

The following communications were made:—

"Proof of the Generality of Certain Formulæ published for a

Special Case by Mr. Blakesley, with Tests of a Transformer." By Prof. W. E. AYRTON, F.R.S., and Mr. J. F. TAYLOR.

"Further Contributions to Dynamometry." By Mr. T. H. BLAKESLEY, M.A.

March 6th, 1891.

Prof. W. E. AYRTON, President, in the Chair.

The following were elected Members of the Society :—

Mr. H. A. MIERS, M.A., and Mr. W. LUCAS, M.A.

The following communication was made :—

"Note on Electrostatic Wattmeters." By Mr. J. SWINBURNE.

Prof. S. P. THOMPSON having taken the Chair,

The following communication was made :—

"Interference with Alternating Currents." By Prof. W. E. AYRTON, F.R.S., and Dr. W. E. SUMPNER.

March 20th, 1891.

Prof. W. E. AYRTON, President, in the Chair.

The following communications were made :—

"The Theory of Dissociation into Ions, and its Consequences." By Mr. S. U. PICKERING, F.R.S.

"Magnetic Proof Pieces and Proof Planes." By Prof. S. P. THOMPSON.

April 17th, 1891.

Prof. W. E. AYRTON, President, in the Chair.

The following communications were made :—

"On a Property of Magnetic Shunts." By Prof. S. P. THOMPSON.

"An Alternating-Current Influence Machine." By Mr. JAMES WIMSHURST.

"On Erecting Prisms for the Optical Lantern." By Prof. S. P. THOMPSON.

May 9th, 1891.

On this occasion the usual meeting gave place to a visit to Cambridge University. At a meeting in the Cavendish Laboratory,

Prof. W. E. Ayrton, President, in the Chair.

the following communications were made:—

"Some Experiments on the Electric Discharge in Vacuum Tubes." By Prof. J. J. Thomson, M.A., F.R.S.

"Some Experiments on the Velocity of Ions." By Mr. W. C. D. Whetham, B.A.

"On the Resistance of some Mercury Standards." By Mr. R. T. Glazebrook, M.A., F.R.S.

"On an Apparatus for Measuring the Compressibility of Liquids." By Mr. S. Skinner, M.A.

"Some Measurements with a Pneumatic Bridge." By Mr. W. N. Shaw, M.A.

May 22nd, 1891.

Prof. W. E. Ayrton, President, in the Chair.

Mr. F. H. Nevill was elected a Member of the Society.

The following communications were made:—

"On Dr. Schobben's Form of Lantern Stereoscope." By Mr. C. J. Woodward.

"On a new Form of Steam-Engine Indicator." By Prof. J. Perry, F.R.S.

"On Mr. Blakesley's Method of Measuring Power in Transformers." By Prof. J. Perry, F.R.S.

June 12th, 1891.

Prof. W. E. Ayrton, President, in the Chair.

Mr. W. H. Dines, B.A., was elected a Member of the Society.

The following communications were made:—

"Alternate Current and Potential Difference Analogies in the Methods of Measuring Power." By Prof. W. E. Ayrton, F.R.S., and Dr. Sumpner.

"On a Clock for Pointing out the Direction of the Earth's Orbital Motion." By Prof. O. Lodge, F.R.S.

"Some Experiments with Leyden Jars." By Prof. O. Lodge, F.R.S.

June 26th, 1891.

Prof. W. E. Ayrton, President, in the Chair.

Mr. J. Enright, B.Sc., was elected a Member of the Society.

The following communications were made:—

"The Construction of Non-inductive Resistances." By Prof. W. E. Ayrton, F.R.S., and Mr. T. Mather.

"On the Influence of Surface-Loading on the Flexure of Beams." By Prof. C. A. Carus-Wilson.

"On Pocket Electrometers." By Prof. C. V. Boys, F.R.S.

"On Electrification due to the Contact of Gases with Liquids." By Mr. J. Enright.

"On the Expansion of Chlorine by Heat." By Dr. Arthur Richardson.

November 6th, 1891.

Dr. E. Atkinson, Vice-President, in the Chair.

The following were elected Members of the Society:—

Miss Alice Lee, Mr. W. A. Shenstone, and Mr. F. McLean.

The following communication was made:—

"On the Generalizations of Van der Waals regarding Corresponding Temperatures, Pressures, and Volumes." By Prof. Sydney Young, D.Sc.

November 20th, 1891.

Prof. W. E. Ayrton, President, in the Chair.

The following communications were made:—

"On the Generalizations of Van der Waals." By Dr. Philippe Guye.

"A new Theory concerning the Constitution of Matter." By Dr. C. V. Burton.

December 4th, 1891.

Prof. W. E. Ayrton, President, in the Chair.

The following were elected Members of the Society:—

Messrs. P. L. Grey, A. Anderson, H. Davey, L. W. Fulcher, H. H. Hoffert, and W. Watson.

The following communications were made:—

"A Permanent Magnetic Field." By Mr. W. Hibbert.

"Note on Rotatory Currents." By Prof. W. E. Ayrton, F.R.S.

"On Struts and Tie-Rods laterally loaded." By Prof. J. Perry, F.R.S.

December 18th, 1891.

Prof. W. E. Ayrton, President, in the Chair.

Mr. R. W. Mond was elected a Member of the Society.

The following communication was made:—

"Note on Interference with Alternating Currents." By Mr. H. Kilgour.

January 22nd, 1892.

Prof. O. Lodge, Vice-President, in the Chair.

The following were elected Members of the Society:—

Mr. J. B. Peace and Mr. E. G. Highfield.

The following communication was made:—

"On the Driving of Electromagnetic Vibrations and Electromagnetic and Electrostatic Engines." By Prof. G. F. FitzGerald, F.R.S.

Annual General Meeting.

February 12th, 1892.

Prof. W. E. Ayrton, President, in the Chair.

The following Report of the Council was read by the Chairman:—

In the year which has elapsed since the last Annual General Meeting of this Society, there have been held 13 Ordinary Meetings in the Laboratory at Kensington, and one Special Meeting at the University of Cambridge. Thus there has been one meeting more than the usual number in one year; and it is the hope of the Council that with an increasing number of communications it will be found possible to multiply in proportion the number of meetings. Many difficulties, however, would have to be overcome before any definite increase—such as weekly meetings instead of only two in a month—can be safely adopted. It is unfortunately the case that Members are in the habit of delaying their communications till the Session is far advanced and the recess is approached. Such a course results in a block of business in June, so that the papers and subjects often fail to receive adequate attention, and at the end of the Session have even to be taken as read.

In the early portion of the Session, on the other hand, there is not unfrequently a smaller supply of communications than is sufficient for the number of meetings held at present.

Again, time could be saved towards the end of the Session by grouping papers of a kindred nature, and discussing them together. If Members would give the Secretaries long notice that they have certain papers in hand, something might be done in this direction; but with the present procedure it is impossible to reap any advantage from this idea.

The special meeting at Cambridge took place on May 9th, 1891. On that occasion the Members of the Society were hospitably entertained by the Colleges of Emmanuel and Trinity, and had the opportunity of inspecting the Cavendish Laboratory and of hearing papers from Members and others resident at the University. The Council feel that the Society has every reason to congratulate itself upon the success of this visit, and think that such visits might be made more frequently.

The Council has to record the deaths of the following Members of the Society:—

Prof. W. Weber, Honorary Member; Dr. W. H. Stone; W. G. Gregory, M.A.; Prof. J. C. Adams, F.R.S.

The Council recommend the name of Professor Van der Waals for election as Honorary Member to fill the vacancy caused by the death of Prof. Weber.

The numbers of the Society now exceed 400.

The following additions have been made to the Library during the year 1891–92:—

Newspapers and Magazines:—

Nature.
The Electrical Review.
Engineering.
The Electrician.
Electrical Plant.
Ingeniero y Ferretero Español y Sud Americano.
Crónica Cientifica.
The Open Court.
The Philosophical Magazine.
Beiblätter der Physik.
Annalen der Physik.
Journal de Physique.
Annales de Chemie et de Physique.
Archives Générales de Médicine.

Journals of Societies, British:—

Proceedings of the Royal Society.
Journal of the Society of Arts.
Journal of the Institute of Electrical Engineers.
Proceedings of the Royal Institution.
Transactions of the Philosophical Society of Cambridge.
Proceedings of the Philosophical Society of Cambridge.
Proceedings of the Manchester Literary and Philosophical Society.
Transactions and Proceedings of the Royal Dublin Society.
Quarterly Journal of the Royal Meteorological Society.
Proceedings of the Institute of Mechanical Engineers.

Journals of Societies, Colonial:—

Journal and Proceedings of the Royal Society of New South Wales.
Mathematical and Physical Society of Toronto University, Papers.

Proceedings and Transactions of the Nova Scotia Institute of Natural Science.

Journals of Societies, Foreign:—

Proceedings of the Academy of Natural Sciences, Philadelphia.
Journal of the Physical and Chemical Society of Russia.
Proceedings of the Physical Society of Berlin.
Physical Society of France. Mémoires.
Physical Society of France. Séances.
Journal of the College of Science of Japan.
Bulletin International de l'Acad. Sci. Cracowie. Séances.

Books, &c.:—

Ohio Meteorological Report.
Geological and Natural History Survey of Canada, vol. iv.
Hong Kong Observatory, Observations.
Metric Measures. Latimer Clark.
North Atlantic Expedition. xx.
The State Weather Service. F. E. Nipher.
St. Louis Engineering Club. President's Address.
Smithsonian Report, 1888.
Studies in Statistics. G. B. Longstaff.
Traité pratique de la Thermométrie de Precision. C. E. Guillaume.
Documents relating to the fixing of a Standard of Time. Ottawa.
Recherches Expérimental Aerodynamiques et données d'Expérience. S. P. Langley.
Three Editorials on Tubercle. Philadelphia.
Wissenschaftliche und Technische Arbeiten. Werner Siemens.
On the Causes of the Phenomena of Terrestrial Magnetism (in English and French), by Henry Wilde.
Catalogue of Books, Radcliffe Library, Oxford, added 1890.
*The Working and Management of an English Railway. G. Findlay.
*The Practical Telegraph Handbook, by S. Poole.
*The First Book of Electricity and Magnetism. W. Perrin Maycock.
Astronomy and Astro-Physics.
Comparative Photographic Spectra of the Sun and Metals. F. McClean.

* Presented by Messrs. Whittaker & Co.

Prof. Wilhelm Eduard Weber, who died in June of last year, at the age of 87, was one of the first of the Honorary Members elected by the Society. It was fitting that this should be so, for though advancing years prevented his taking an active part in the most recent progress of Physics, he was one of those who had the chief share in laying the foundations on which modern conceptions in one of the most important branches of our science rest. He was not, like Faraday, a discoverer of new phenomena by which whole new territories were added to the domain of science; his work was rather that of systematizing, ascertaining the exact mutual bearings of phenomena, and establishing permanent landmarks whereby all after-comers might guide their ways and pass swiftly and easily over what was before a confused and difficult region.

In the Royal Society Catalogue more than sixty papers on various branches of Physics are enumerated of which W. E. Weber was the author. About twenty of these, dating from 1825 to 1835, relate, with very few exceptions, to questions of acoustics; the remainder are concerned with magnetism and electricity. There can be no doubt that the direction which Weber's scientific activity took from about 1835 onwards was determined by his having been, as Professor of Physics at Göttingen, the colleague and associate of Gauss. For seven years (1836 to 1842) they cooperated in directing the work of the *Magnetische Verein* and in issuing the important series of volumes embodying the '*Resultate aus den Beobachtungen*' of this association.

The great work of Weber's life was the establishment on a firm basis of the *absolute* system of measurement for electrical magnitudes. His first step in this direction seems to have been the absolute measurement of the strength of an electric current in 1840. Weber's first definition of the unit strength of current, and the experimental method by which he measured actual currents in terms of this unit, were founded directly on Gauss's definition and method of measuring the absolute strength of a magnetic field. From the measurement of currents he went on to the measurement of electro-chemical equivalents, electromotive forces, and resistances. One very important piece of work was the verification of Ampère's law of the mutual action between two currents, and the exact determination of the equivalence between electric circuits and magnets.

As one result of this investigation, Weber showed how the definition and measurement of the strength of currents may be founded upon the observation of the forces exerted between two

parts of the same current, without any reference to the unit of magnetic force. He also showed that a corresponding definition of the unit of electromotive force may be given, and consequently that electrical resistance may be measured in absolute units independently of the determination of the strength of a magnetic field. His last published work seems to have been a determination of absolute resistance, carried out in conjunction with F. Zöllner in 1880.

In connexion with recent investigations relating to the propagation of electrical vibrations, it is interesting to find Weber remarking, in 1846, that the laws of electrodynamic force would not unlikely be susceptible of simpler expression, if account were taken of an intervening medium, than is possible when the phenomena are considered from the point of view of action at a distance; and, further, that it is not improbable that the medium concerned in electric phenomena may be the luminiferous æther. Again, in a paper in 1864 on Electric Vibrations, he determined their velocity of propagation as $3{\cdot}107 \times 10^{10}$ centim. per second, and pointed out that this is the velocity of light, observing that Kirchhoff had previously come to the same result and drawn attention to the same coincidence.

With regard to Weber's experimental work, it may be said that no better models of careful and accurate work and clear statement of results obtained can to this day be put in the hands of a student of Physics than his and Gauss's papers in the *Resultate aus den Beobachtungen des magnetischen Vereins* and Weber's *Elektrodynamische Maassbestimmungen.*

Weber was born at Wittenberg in 1804; he studied at the University of Halle, where he became Professor-Extraordinary of Physics in 1828. He received a call to the Chair of Physics at Göttingen in 1831, but he was ejected from it, on political grounds, in 1837. From 1843 to 1849, when he returned to Göttingen, he was Professor of Physics at Leipzig. He had two brothers—Ernst Heinrich (born in 1795, died 1878), Professor of Anatomy and Physiology at Leipzig, in conjunction with whom he published, in 1825, the celebrated work on Wave-Motion (*Die Wellenlehre auf Experimente gegrundet*); and a younger brother, Eduard Friedrich, who was a distinguished anatomist, and was for many years prosector in the University of Leipzig. W. E. Weber was elected a Foreign Member of the Royal Society in 1850; his brother Ernst received the same honour in 1862.

WALTER GEORGE GREGORY (the youngest son of George J. G. and Eliza Gregory, Eglesfeld House, Dorchester, Dorset) was born on January 31st, 1859. He commenced his education at the Dorchester Grammar School, afterwards going to Weymouth College, and then to Bristol Grammar School, whence, in 1878, he obtained an open Mathematical Scholarship at Queen's College, Oxford. He took his B.A. degree in Mathematics in June 1882, obtaining two First Classes, and then worked for a year at the Clarendon Laboratory. On leaving Oxford, in 1883, he obtained the appointment of Demonstrator in Physics at the Royal Indian Engineering College, Coopers Hill, which appointment he retained till within two months of his death, which occurred on November 25, 1891.

Never very strong, the last years of his life were spent in a constant struggle against ill-health, which left him little superfluous energy to devote to work outside the ordinary routine. At the same time, it was a matter of surprise to those that knew him that he managed to do what he did. He was devoted to his subject, and practically lived in it; clear-headed in a remarkable degree, he could take in very quickly the conditions of a physical problem, and follow up its side issues, and consequently could often make most valuable suggestions to others in their work. He had a decided liking for mechanism and practical applications of scientific facts from his earliest years, and was constantly working at some invention or other of his own. He was an invaluable colleague in a Physical Laboratory in the designing of apparatus and doing anything requiring delicate manipulation. He contributed two papers to the Physical Society in November 1889, "On a Method of Driving Tuning-Forks Electrically," and "On a New Electric Radiation Meter," which furnish a slight indication of what he might have done with health and strength.

JOHN COUCH ADAMS was born at Lidcot, near Launceston, in Cornwall, on June 5, 1819. He received his early education at the village school and at Devonport, where he gave evidence of his remarkable faculty for mathematical and astronomical study. In October 1839 he entered at St. John's College, Cambridge; and in 1843 he graduated as Senior Wrangler and first Smith's Prizeman, becoming shortly afterwards a Fellow and tutor of his College.

Both before and after taking his degree he was fascinated by a problem which was at that time profoundly interesting astronomers —the irregularities shown by the planet Uranus in its motion. Its

orbit differed from the elliptic path which an undisturbed planet would have pursued; and as the deviations could not be explained by the influence of the other known planets, it was supposed that there must be a more remote planet which had not been observed. To the search for this unknown planet Adams devoted all the energies of his mathematical genius, and everyone knows the brilliant success with which his labours were crowned. His solution was communicated to Prof. Challis in September 1845, and to the Astronomer-Royal in the following month. We need only refer to the facts that similar work was done in 1846 by Leverrier; that the French astronomer's results, unlike those of the English investigator, were at once made known; and that on September 23, 1846, the planet Neptune was found by Dr. Galle, of Berlin, on the basis of Leverrier's elements. Adams and Leverrier rank as joint discoverers, and, as such, they received on February 11, 1848, the gold medal of the Royal Astronomical Society. Some members of Adams's college, in order to mark their sense of the importance of his achievement, raised a fund, which the University accepted, for the founding of a prize, to be called "The Adams Prize," to be awarded every two years to the author of the best essay on some subject of pure mathematics, astronomy, or other branch of natural philosophy. In 1851 he was elected President of the Royal Astronomical Society.

As he did not take orders, his Fellowship at St. John's expired in 1852, but he continued to reside in the College until 1853, when he was elected to Pembroke. In 1858 he was appointed Professor of Mathematics at the University of St. Andrews, but he held this office only during a single session. He became the Lowndean Professor of Astronomy and Geometry, at Cambridge, in 1859, in succession to the late Prof. Peacock, and retained this position during the remainder of his life.

Meanwhile, he had been carrying on many important investigations; and, until ill-health disabled him, his labours were never seriously interrupted. Foremost among his later achievements were the results of his researches on the Moon and on the theory of the November meteors. In 1866 the Royal Astronomical Society awarded him its gold medal for his lunar researches. He had succeeded Prof. Challis as Director of the Cambridge Observatory in 1861; and in 1884 he served as one of the delegates for Great Britain at the International Meridian Conference at Washington.

For about a year and a half before his death Prof. Adams was

too ill to do as much work as he had been accustomed to do, and during the last ten weeks he was confined to bed. He died on the morning of January 21.

He was a Fellow of the Royal Society, and of the leading foreign scientific bodies; and honorary degrees were conferred upon him by his own University and by Oxford. The post of Astronomer-Royal was offered to him by the First Lord of the Admiralty in 1881, on Sir George Airy's retirement, but declined by him on the ground of age.

The adoption of the Report was proposed and carried unanimously.

The Society then elected Professor Van der Waals an Honorary Member of the Society.

The election of Officers and other Members of Council then took place, the new Council being constituted as follows :—

President.—Prof. G. F. Fitzgerald, M.A., F.R.S.

Vice-Presidents who have filled the Office of President.—Dr. J. H. Gladstone, F.R.S.; Prof. G. C. Foster, F.R.S.; Prof. W. G. Adams, M.A., F.R.S.; Lord Kelvin, D.C.L., LL.D., P.R.S.; Prof. R. B. Clifton, M.A., F.R.S.; Prof. A. W. Reinold, M.A., F.R.S.; Prof. W. E. Ayrton, F.R.S.

Vice-Presidents.—Prof. A. W. Rücker, M.A., F.R.S.; Walter Baily, M.A.; Prof. O. J. Lodge, D.Sc., F.R.S.; Prof. S. P. Thompson, D.Sc., F.R.S.

Secretaries.—Prof. J. Perry, D.Sc., F.R.S.; T. H. Blakesley, M.A., M.Inst.C.E.

Treasurer.—Dr. E. Atkinson.

Demonstrator and Librarian.—C. Vernon Boys, F.R.S.

Other Members of Council.—Shelford Bidwell, M.A., LL.B., F.R.S.; W. E. Sumpner, D.Sc.; Major-General E. R. Festing, R.E., F.R.S.; J. Swinburne; Prof. J. V. Jones, M.A.; Rev. F. J. Smith, M.A.; Prof. W. Stroud, D.Sc.; L. Fletcher, M.A., F.R.S.; G. M. Whipple, D.Sc.; James Wimshurst.

Votes of thanks were passed to the Lords Committee of the Council on Education; to the Officers; and to the Auditors.

THE TREASURER IN ACCOUNT WITH THE PHYSICAL SOCIETY, FROM JANUARY 1ST, 1891, TO DECEMBER 31ST, 1891.

Dr.	£ s. d.	£ s. d.
Balance in Bank, January 1, 1891		309 19 10
Life Subscriptions	120 0 0	
Entrance-Fees	23 0 0	
Subscriptions for 1888	2 0 0	
" 1889	5 0 0	
" 1890	14 0 0	
" 1891	109 0 0	
Excess Subscriptions	1 2 0	
Prepaid Subscriptions:—1892	7 0 0	
" 1893	1 0 0	
		282 2 0
Paid to Treasurer for copy of Joule's papers		0 16 0
One year's Dividend on £400 Furness 4 per cent. Debenture Stock, less Income Tax	15 12 0	
One year's Dividend on £575 Midland 4 per cent. Preference Stock, less Income Tax	22 8 6	
One year's Dividend on £200 Lancaster Corporation Stock, less Income Tax	7 16 0	
Five Quarters of a year's Dividend on £200 Metropolitan Board of Works Stock, less Income Tax	8 10 10	
One year's Dividend on £254$\frac{2}{3}$ New South Wales 3½ per cent. Stock, less Income Tax	8 13 6	
		63 0 10
Taylor and Francis		32 14 7
Balance due to Treasurer		1 0 0
		£689 13 3

Cr.	£ s. d.	£ s. d.
Balance due to Treasurer		9 18 10
Excess Subscriptions returned		1 2 0
" received in 1890		2 0 0
Taylor and Francis:—		
Proceedings, vol. ii. (part 1)	32 9 0	
Postages, &c.	4 4 6	
Members' separate copies	6 16 6	
Miscellaneous printing	18 18 5	
		62 8 5
Haggard (Binding)		5 0 5
Williams and Norgate		4 7 0
Gyde (Binding)		3 0 0
Attendance at Meetings		7 2 2
Reporting		21 4 0
Barth, Royalty on Van der Waals' "Continuity of the Gaseous and Liquid States"		5 0 0
Bank charges		1 1
Petty Cash:—		
Mr. Blakesley	1 18 0	
Professor Rücker	2 1 0	
		3 19 0
Cheques not paid in		12 0 0
Balance in Bank		552 10 4
		£689 13 3

Audited and found correct,

HENRY M. ELDER, } *Auditors.*
ALFRED H. FISON,

February 3rd, 1892.

PROPERTY ACCOUNT OF THE PHYSICAL SOCIETY, December 31, 1891.

Assets.	£	s.	d.	Liabilities.	£	s.	d.
Balance in Bank	552	10	4	Balance due to Treasurer	1	0	0
Subscriptions due	180	0	0	Subscriptions anticipated	8	0	0
£400 Furness 4 per cent. Debenture Stock	456	0	0				
£575 Midland 4 per cent. Preference Stock	650	0	0				
£200 Metropolitan Board of Works Stock	217	0	0				
£200 Lancaster Corporation Stock	224	0	0				
£$254\frac{3}{8}$ New South Wales $3\frac{1}{2}$ per cent. Stock	249	7	5				
Cheques unpaid	12	0	0	Balance	2531	17	9
	£2540	17	9		£2540	17	9

Audited and found correct,

A. H. FISON,
HENRY M. ELDER, } *Auditors.*

February 3rd, 1892.